IMPUTATION METHODS ON GENERAL LINEAR MIXED MODELS OF LONGITUDINAL STUDIES

YANN-YANN SHIEH
AMERICAN INSTITUTES FOR RESEARCH
1000 THOMAS JEFFERSON ST. NW. WASHINGTON DC 20007. YSHIEH@AIR.ORG

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Abstract: Survey data collection is a very efficient way to gather information for research of interest. Many state and federal government agencies collect data through surveys. Long-term longitudinal studies are the most appropriate studies for the study of individual change over time. As a result, most longitudinal studies and surveys have non-responses and missing data issues. The problem of missing data arises frequently in practice in applied research settings. Imputation is a way to handle missing data. General linear mixed models are commonly used in the analysis of unbalanced repeated measures designs (Verbeke & Molenberghs, 2000). In this study, we simulate longitudinal data under a variety of missing data patterns that is contrasted with modeling using missing value imputation methods in the use of general linear mixed modeling of longitudinal surveys with the focus of documenting the characteristics of the model parameter estimates and their standard errors.

1. INTRODUCTION

Survey data collection is a very efficient way to gather information for research of interest. Many state and federal government agencies collect data through surveys. Some studies are often designed to investigate changes in a specific parameter, which is measure repeatedly over time. Long-term longitudinal studies are the most appropriate studies for the study of individual change over time and factors likely to influence change over time. As a result, most longitudinal studies and surveys have non-responses and missing data issues. Missing or incomplete data are a serious problem in many fields of research for data analysis, leaving the question: How to handle the missing values in such a way to make the result be as close to the truth as possible?

One approach is to analyze cases with complete data. Another is to use of complete-data statistics on data sets with missing values filled-in. Alternatively, one can used maximum-likelihood approaches, such as that in general linear mixed modeling, to deal with missing data. Imputation is the name given to any method whereby missing values in a data set are filled-in with plausible estimates. There are several methods for imputing the missing values. In some cases, when auxiliary information is properly used, imputation increases statistical accuracy. A key point that is clear from the missing data literature is to choose a computational method or combination of methods based on the nature of the problem, the computational resources, the accuracy requirement, and the degree of difficulty of any required theoretical derivations.

The purposes of this study are the followings. First, we simulate longitudinal data with different missing data patterns and examine the impact of missing data pattern on the quality of parameter estimates and their standard errors. Second, we select examples for some of these unbalanced missing data designs– but focus primarily on a case study where observation times are fixed but data are missing at some of the time points, to impute the missing data and compare the results with and without imputed values using general linear mixed modeling of data sets.

2. GENERAL THEORY AND METHODOLOGY

2.1 General linear mixed model

The data analyses will be expressed in the general linear mixed model family: $\mathbf{Y}_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{Z}_i\mathbf{b}_i + \boldsymbol{\varepsilon}_i$, for $i = 1,...,n$ independent units, where $b_i \approx N(\mathbf{0}, \mathbf{D})$, $\varepsilon_i \approx N(\mathbf{0}, \boldsymbol{\Sigma})$, and $\mathbf{b}_i$ and $\boldsymbol{\varepsilon}_i$ are statistically independent, $\mathbf{Y}_i$ is the ($\tau_i \times 1$) response vector. $\mathbf{X}_i$ is a ($\tau_i \times p$) design matrix for the fixed effects. $\boldsymbol{\beta}$ is a ($p \times 1$) vector of unknown fixed regression coefficients. $\mathbf{Z}_i$ is a ($\tau_i \times q$) design matrix for the random effects. Though a number of estimation strategies are available, the current paper uses maximum likelihood estimation as implemented in the general linear mixed modeling procedure in HLM.

2.2 Missing value mechanisms and pattern

There are a number of different ways to conceptualize how missing data arises. Little and Rubin (1987) introduced specific missing data terminology as a standard framework to deal with missing data mechanisms and their effect on data analysis. Little and Rubin (1987) found it useful to distinguish between data that are Missing Complete at Random, Missing at Random, and Non-ignorable Missing, where:

(1) Missing Completely at Random (MCAR). If the probability that a response is missing is independent of both the observed data for that case and the unobserved responses are simple a random sample from the observed data. An example of MCAR missing data arises when investigators randomly assign research participants to complete two-thirds of a survey instrument. Graham, Hofer, MacKinnon (1996) illustrate the use of planned missing data patterns of this type of gather responses to more survey items from fewer research participants than one ordinarily obtains from the standard survey completion paradigm where every research participant receives and answers each survey question.
(2) Missing at Random (MAR). If the probability that a response is missing depends on the observed data, but not on the unobserved data. This assumes the parameters of the model for the data are distinct from the parameters of the missingness mechanism. The missingness mechanism is ignorable. For example, in a reading comprehension test at the beginning of a survey administration session, research participants with lower reading comprehension scores may be less likely to complete the entire survey. The missing data are due to some other external influence.
(3) Non-ignorable Missing. When respondents and non-respondents, with the same values of some variables observed for both, differ systematically with respect to values of the variable missing for the non-respondent. In other words, the pattern of data missingness is non-random and it is not predictable from other variables in the database. For example, a participant in a weight-loss study does not attend a weigh-in due to concerns about his/her weight loss, his/her data are missing due to non-ignorable factors.

In practice it is usually difficult to meet the MCAR assumption. MAR is an assumption that is more often but not always tenable. Ignorability is a judgment made by the data analyst. Rubin (1976) addressed the problem of missing data. He mentioned that when making sampling distribution inferences about the parameter of the data, it is appropriate to ignore the process that causes missing data if the missing data are missing at random and the observed data are observed at random, but the inferences are generally conditional on the observed pattern of missing data. Rubin (1976) farther suggested that when dealing with real data, data analyst or statistician should explicitly consider the process that causes missing data and needs models for the process.

2.3 General theory for imputation

Two principal approaches to estimation with missing data are weighting and imputation. Weighting typically is used in unit nonresponse which is viewed as the inverse of the response probabilities associated with the response mechanism. Imputation is used in item nonresponse. The imputed values are sample-based. There are a variety of imputation methods. The goal of any imputation technique is to produce a complete data set, which can then be analyzed using complete-data inferential method. The observed values are used to impute values for the missing observations. Two kinds of imputation methods are discussed: Single Imputation and Multiple Imputation.

Single Imputation
(1) Mean Imputation: The sample mean of a variable replaces any missing data for that variable.
(2) Hot-deck Imputation: Missing values are replaced with values taken from matching respondents.
(3) Last Value Carried Forward or LVCF: The last observed value is used to fill in missing values at subsequent points in a longitudinal study.
(4) Predicted Mean: An ordinary least-squares multiple regression algorithm is used to impute the most likely value. In this method, researchers develop a regression equation based on complete case data for a given variable, treating it as the outcome and using all other relevant variables as predictors.

Single imputation is easy to employ with a single value imputed for a missing value. However, there are disadvantages of single imputation since it does not reflect extra uncertainty and does not display variation due to missing data. Rubin (1986) sees a disadvantage of single imputation "…the one imputed value cannot in itself represent uncertainty about which value to impute: If one value were really adequate, then that value was never missing. Hence, analyses that treat imputed values just like observed values generally systematically underestimate uncertainty, even assuming the precise reason for nonresponse are known."

Multiple Imputation

Multiple Imputation, first proposed by Rubin in the early 1970's (Rubin, 1976) as a way to address survey non-response and issues associated with single imputation, involves replacing each missing values by M (M>=2) imputed values to create M complete data sets. Multiple imputation carries out analysis under each set of imputation and combines analyses to reflect within-imputation and between-imputation variability. Several techniques involved in multiple imputation are mentioned by Rubin (1986), Little and Rubin (1987), Schafer and Olsen, (1998), and Schafter (1999). In the current study, we use two multiple imputation methods as follows.

(1) Predictive Model Based Method: An ordinary least-squares regression method of imputation is used. Model is estimated from the observed data, then using this estimated model, a new linear regression parameters are randomly drawn from their Bayesian posterior distribution.
(2) Propensity Score: An implicit model approach based on Propensity Scores and an Approximate Bayesian Bootstrap is used to generate the imputations.

Multiple imputation has its advantage and disadvantage. The major advantages of multiple imputation as indicated by Rubin (1986) are that standard complete-data methods are used to analyze each completed data set; moreover, the ability to utilize data collector's knowledge in handling the missing values is not only retained but actually enhanced. In addition, multiple imputations allow data collectors to reflect their uncertainty as to which values to impute. Disadvantages include the time intensiveness imputing five to ten data sets, testing models for each data set separately, and recombining the model results into one summary.

3. SIMULATION STUDY– LONGITUDINAL GROWTH DATA WITH MISSING DATA PATTERNS

The data sets under consideration for the present paper are derived from National Education Longitudinal Study of 1988 (NELS:88) to obtain the parameters for simulation. NELS:88 is the most current and comprehensive source of information on personal and contextual factors in the educational life of U.S. adolescents over time. It began in 1988 with a cohort of 25,000 eighth-graders and follow-up data were collected in 1990, 1992, 1994, and 2000. We used students math achievement scores to simulate missing data under different missing data mechanism.

3.1 Simulated model

Data were simulated from two-level models with either four or eight waves. These models describe an increasing linear trend in achievement for students being tested on a standard test, administered each semester, over a two- or four-year period. Each student (denoted by i) has an 'ability' latent variable written as a_i which remains constant over the testing periods. On test period t (t=1, …, T) the test score for the i-th student is $y_{it..}$ The mean test score for student i is a linear function of time and student's ability $a_{i:}$ $\mu_{it} = b_0 + b_1 t + a_i$

The actual test score on time period t has an random errors e_{it}, so that $y_{it} = u_{it} + e_{it}$

The ability distribution for the a_i has a mean of 0 and variance of $\sigma^2 a$ (i.e., N(0,$\sigma^2 a$)) and the error distribution for the e_{it} also has mean of 0 and variance of $\sigma^2 e$ (i.e., N(0,$\sigma^2 e$)). These are standard assumptions for the two-level model level model.

3.2 Time waves

Two longitudinal time waves were examined in this study (T = 4 and T = 8). In addition, we modeled the linear trend regression over time as different for boys and girls. For T = 4 the regression for boy was 50+10t and for girls it was 35+15t, so that the boys started at score 50 at t = 1 and increased to score 90 at t = 4, while girls started at score 35 at t=1 and increased to score 95 at t = 4. For T = 8, the intercepts were assumed the same and the sloped were halved, giving the same initial and final means. Therefore, the data were generated from an interaction model including a dummy variable g for gender (g=0 for boys, g=1 for girls). $y_{it} = 50 + 10t - 15g_i + 5g_i * t + a_i + e_{it.}$

3.3 Missing data mechanisms, and probabilities of missingness

Two missing data patterns were used in this study:

(1) MCAR-missing completely at random. In this case, any test score is missing independently of the others with a constant probability p over the four or eight time periods of either 0.05 or 0.1. The proportions of complete observations with all scores are for T = 4, 0.81 for p = 0.05 and 0.66 for p = 0.1, and for T = 8, 0.66 for p = 0.05 and 0.43 for p = 0.1.
(2) MAR-missing at random. Any test score is missing independently of the others, but with a probability p_t which increases with time: p_t = 0.0, 0.025, 0.05, and 0.075 for T = 4, and p_t = 0.0, 0.0125, 0.025, 0.0375, 0.05, 0.0625, 0.075, and 0.0875 for T = 8.

3.4 Data generation procedures

A two-stage sampling procedure for generating simulation data was used. Two types of data were examined: full data and complete data. We generated the T values of y_{it} from the model for each student i , and generated a corresponding set of dummy indicators d_{it} from the missingness model, where d_{it} = 0 if the corresponding test score y_{it} is to be missing (with probability 1-p or 1-p_t) and d_{it} = 1 if y_{it} is to be observed (with probability 1-p or 1-p_t). The 'full data' for case I consists of the set of $\sum_t d_{it}$ responses. If all d_{it} for this case i is 1, the response is complete and the case is appended to the 'complete case' data set, otherwise, it is omitted as incomplete. Therefore, the 'full data' set consists of n strings of between 1 and 4 or 1 and 8 responses. The 'complete case' data set consists of less than n strings of length 4 and 8.

3.5 Parameter estimation

To be able to make better comparisons between parameter values of different magnitude, the following quantities for each data set were computed:
(1) The average values of the estimated parameters (both fixed –regression coefficient, and random-variance components).
(2) The bias of estimated parameters, by subtracting the true values of the parameters.
(3) The standard errors of the parameter estimates.

4. STUDY EXAMPLES FOR IMPUTATION AND ANALYSIS STRATEGY

For the current paper, based on different missing data mechanisms and patterns, four incomplete data sets are selected for imputation from the above incomplete data set. These incomplete data sets are described in the following section.

4.1 Incomplete data sets

Data Subset 1 (MCAR). We assume 500 students taking tests in the four continuous semesters. Some students missed test scores.
Data Subset 2 (MAR). We assume 500 students taking tests in the four continuous semesters. Some students missed test scores.
Data Subset 3 (MCAR). We assume 500 students taking tests in the eight continuous semesters. Some students missed test scores.
Data Subset 4 (MAR). We assume 500 students taking tests in the eight continuous semesters. Some students missed test scores.

4.2 Imputation data sets

Currently a number of statistical software packages and procedures are available to impute missing values. In this paper, we use Solas (Software for Missing Data Analysis 3.2) software. The imputation data sets are labeled as a function of the data subset on which the imputations were made (data subset 1, 2, 3, or 4) and as a function of the imputation method used to fill-in missing values: a) No mputation, b) Hot-deck Imputation , c) Group Mean Imputation, d) Last Value Carried Forward (LVCF), e) Predicted Mean f) Multiple imputation based on M imputations using multiple regression analysis, g)M=c (Multiple imputation based on M imputations using Propensity Scores and an Approximate Bayesian Bootstrap approach). For each data set a range of M values were used. Due to space considerations results on only some of the M values are reported.

4.3 Parameter estimation

A general linear mixed model approach using maximum likelihood estimation (HLM program) was used for all the data sets under consideration. Parameter estimates and standard errors were obtained for each study data set parameter. In the case of analyses of incomplete and single-value imputation method data sets, a single set of results were obtained. In the case of multiple imputation, where *M*=*c*, *c* sets of results were obtained and pooled as follows to generate the *M*=*c* results reported.
The *M* within-imputation estimates for θ (the parameter of the interest) are pooled to give the multiple imputation estimates: $\hat{\theta}* = M^{-1}\sum_{m=1}^{M}\hat{\theta}^{(m)}$. Now, suppose that complete data inference about θ would be made by $(\theta-\theta^*) \sim N(\mathbf{0},\mathbf{Y})$. Then, one can make normal based inferences for θ based upon $(\theta-\theta^*) \sim N(\mathbf{0},\mathbf{V})$, where $V = \hat{W} + M^{-1}(M+1)\hat{B}$, such that

$$\hat{W} = M^{-1}\sum_{m=1}^{M}U^{(m)}$$

is the average within-imputation variance, and

$$\hat{B} = (M-1)^{-1}[\sum_{m=1}^{M}(\hat{\theta}^{(m)} - \hat{\theta}^{*})(\hat{\theta}^{(m)} - \hat{\theta}^{*})']$$

is the between-imputation variance.

5. RESULTS AND SUMMARY

Across the series of analyses conducted, the following results were observed:

5.1 Parameter estimates for missing data set

The average parameter estimates for complete data set and full data set across examined conditions are shown in the table 1. In looking at Table 1, both complete cases and full data estimates are consistent, as expected under theory, for both MCAR and MAR missingness patterns. For the bias parameter estimates and standard errors associated with the parameter estimates, when sample size increased, the bias and standard errors decreased as expected. However, biases were much smaller than standard errors (see Tables 2 and 3). In general, the biases of the complete cases estimates were larger and their standard errors were consistently larger than those of the full data estimates.

Table 1
Average Parameter Estimates by Examined Data Sets and Conditions

		Complete case						Full case					
Condition	N	γ_{00}	γ_{10}	γ_{01}	γ_{11}	σ_0^2	σ_ε^2	γ_{00}	γ_{10}	γ_{01}	γ_{11}	σ_0^2	σ_ε^2
SIZE													
500	16	49.967	7.501	-14.939	3.746	59.169	40.013	50.016	7.502	-14.967	3.741	59.395	40.256
1000	16	49.986	7.497	-14.966	3.743	59.596	40.032	50.017	7.498	-15.012	3.743	59.703	40.126
2000	16	50.016	7.491	-14.977	3.743	59.907	40.134	50.032	7.497	-15.007	3.748	60.122	40.161
ICC													
0.3	12	50.006	7.499	-14.957	3.738	29.766	70.125	50.026	7.498	-14.992	3.742	29.805	70.281
0.5	12	49.962	7.490	-14.964	3.745	49.502	50.044	50.025	7.499	-14.995	3.743	49.675	50.309
0.7	12	49.998	7.498	-14.963	3.746	69.535	30.052	50.019	7.499	-14.994	3.744	69.756	30.111
0.9	12	49.992	7.499	-14.958	3.747	89.427	10.018	50.017	7.500	-15.002	3.747	89.725	10.022
TIME													
4	24	50.083	9.998	-15.012	4.987	60.131	40.152	50.073	9.998	-15.003	4.988	60.158	40.312
8	24	49.896	4.995	-14.909	2.501	58.984	39.967	49.971	5.000	-14.988	2.500	59.322	40.050
MISSING NESS													
MACR	24	49.974	7.496	-14.945	3.741	59.417	40.049	50.019	7.500	-14.996	3.744	59.743	40.182
MAR	24	50.005	7.497	-14.976	3.748	59.698	40.071	50.024	7.498	-14.995	3.745	59.737	40.179

Table 2
Average Bias Parameter Estimates by Examined Data Sets and Conditions

		Complete case						Full case					
Condition	N	γ_{00}	γ_{10}	γ_{01}	γ_{11}	σ_0^2	σ_ε^2	γ_{00}	γ_{10}	γ_{01}	γ_{11}	σ_0^2	σ_ε^2
SIZE													
500	16	-0.033	0.001	0.061	-0.004	-0.831	0.013	0.016	0.002	0.033	-0.009	-0.605	0.256
1000	16	-0.014	-0.003	0.034	-0.007	-0.404	0.032	0.017	-0.002	-0.012	-0.007	-0.297	0.126
2000	16	0.016	-0.009	0.023	-0.007	-0.093	0.134	0.032	-0.003	-0.007	-0.002	0.122	0.161
ICC													
0.3	12	0.006	-0.002	0.043	-0.012	-0.234	0.125	0.026	-0.002	0.008	-0.008	-0.195	0.281
0.5	12	-0.038	-0.010	0.036	-0.005	-0.498	0.044	0.025	-0.001	0.006	-0.007	-0.326	0.309
0.7	12	-0.002	-0.002	0.037	-0.004	-0.465	0.052	0.019	-0.001	0.006	-0.006	-0.244	0.111
0.9	12	-0.008	-0.001	0.042	-0.003	-0.573	0.018	0.017	-0.001	-0.002	-0.003	-0.275	0.022
TIME													
4	24	0.083	-0.002	-0.012	-0.013	0.131	0.152	0.073	-0.002	-0.003	-0.012	0.158	0.312
8	24	-0.104	-0.005	0.091	0.001	-1.016	-0.033	-0.030	0.000	0.012	0.000	-0.679	0.050
MISSING NESS													
MACR	24	-0.026	-0.004	0.055	-0.010	-0.583	0.049	0.019	0.000	0.004	-0.007	-0.257	0.182
MAR	24	0.005	-0.003	0.024	-0.003	-0.302	0.071	0.024	-0.002	0.005	-0.006	-0.263	0.179

Table 3
Average Standard Errors of Parameter Estimates by Examined Data Sets and Conditions

		Complete case						Full case					
Condition	N	γ_{00}	γ_{10}	γ_{01}	γ_{11}	σ_0^2	σ_ε^2	γ_{00}	γ_{10}	γ_{01}	γ_{11}	σ_0^2	σ_ε^2
SIZE													
500	16	0.796	0.135	1.048	0.186	4.829	1.518	0.672	0.124	0.901	0.178	4.111	1.433
1000	16	0.569	0.100	0.724	0.124	3.804	1.025	0.485	0.091	0.623	0.116	3.040	0.906
2000	16	0.398	0.068	0.520	0.114	2.574	0.717	0.338	0.063	0.434	0.084	2.231	0.616
ICC													
0.3	12	0.593	0.142	0.774	0.218	2.321	1.921	0.510	0.128	0.668	0.175	2.027	1.721
0.5	12	0.580	0.117	0.743	0.156	3.134	1.345	0.501	0.109	0.653	0.148	2.759	1.262
0.7	12	0.581	0.091	0.748	0.121	4.327	0.807	0.493	0.084	0.646	0.115	3.473	0.717
0.9	12	0.596	0.054	0.791	0.071	5.161	0.274	0.487	0.049	0.645	0.066	4.249	0.240
TIME													
4	24	0.595	0.142	0.761	0.205	3.573	1.165	0.552	0.137	0.701	0.187	3.281	1.181
8	24	0.580	0.060	0.767	0.078	3.899	1.009	0.445	0.048	0.606	0.065	2.973	0.789
MISSING NESS													
MACR	24	0.620	0.104	0.817	0.153	4.033	1.121	0.503	0.093	0.657	0.127	3.108	0.977
MAR	24	0.555	0.098	0.711	0.130	3.439	1.052	0.494	0.092	0.649	0.125	3.146	0.993

5.2 Parameter estimates for imputed data sets

Parameter estimates and standard errors were obtained for each example data set and each imputation data set. Tables 4, 5 and 6 displayed the results. In looking at the Table 4, all the imputation data sets produced similar results. Compared the results from Table 4 to Tables 5 and 6, in general, the standard errors from the analysis of the imputation data sets were greater than from the no imputation data sets. For the parameter estimates, the intercepts for the imputation data sets were greater than those from the no imputation data sets; but the slopes were less than those from the no imputation data sets. For the variance component σ_ε^2, predicted mean, a single imputation method, tended to produce smaller values than those from other imputation methods and was less than those from no imputation data sets.

Table 4. Parameter estimates and standard errors for underlying no imputation data sets

Missing Dataset	SIZE	ICC	TIME	MISSING	γ_{00}	γ_{10}	γ_{01}	γ_{11}	σ_0^2	σ_ε^2
Parameter estimates										
complete	500	0.5	4	MACR	50.147	10.012	-15.022	4.980	49.873	50.216
complete	500	0.5	4	MAR	50.161	10.002	-15.041	4.988	49.983	50.228
complete	500	0.5	8	MACR	49.354	4.994	-14.886	2.510	47.950	49.378
complete	500	0.5	8	MAR	49.854	4.994	-14.886	2.510	48.870	49.878
full	500	0.5	4	MACR	50.133	10.007	-15.025	4.978	49.839	50.626
full	500	0.5	4	MAR	50.157	9.999	-15.033	4.980	49.671	50.730
full	500	0.5	8	MACR	49.898	5.000	-14.914	2.501	48.839	50.276
full	500	0.5	8	MAR	49.898	5.000	-14.914	2.501	48.839	50.276
SE										
complete	500	0.5	4	MACR	0.855	0.220	1.119	0.324	4.262	2.103
complete	500	0.5	4	MAR	0.811	0.230	1.098	0.324	4.184	2.152
complete	500	0.5	8	MACR	0.709	0.086	0.888	0.105	3.956	1.589
complete	500	0.5	8	MAR	0.709	0.086	0.888	0.105	3.956	1.589
full	500	0.5	4	MACR	0.766	0.215	1.015	0.318	3.826	2.014
full	500	0.5	4	MAR	0.765	0.218	1.037	0.322	4.359	2.860
full	500	0.5	8	MACR	0.577	0.075	0.783	0.098	3.358	1.330
full	500	0.5	8	MAR	0.577	0.075	0.783	0.098	3.358	1.330

Table 5. Parameter estimates, underlying data subset, and imputation method

SIZE	ICC	TIME	MISSING	Imputation	γ_{00}	γ_{10}	γ_{01}	γ_{11}	σ_0^2	σ_ε^2
500	0.5	4	MACR	Hot-deck	52.403	9.259	-16.384	5.149	61.456	53.224
500	0.5	4	MAR	Hot-deck	51.550	9.474	-17.366	5.435	47.334	52.145
500	0.5	8	MACR	Hot-deck	51.388	4.803	-16.396	2.617	56.704	52.081
500	0.5	8	MAR	Hot-deck	50.657	4.972	-16.171	2.446	49.819	50.701
500	0.5	4	MACR	Group mean	52.877	9.112	-16.110	5.163	52.638	55.286
500	0.5	4	MAR	Group mean	52.148	9.158	-17.529	5.589	44.384	56.971
500	0.5	8	MACR	Group mean	53.433	4.421	-15.503	2.478	43.631	68.896
500	0.5	8	MAR	Group mean	51.198	4.815	-15.974	2.379	45.153	58.984
500	0.5	4	MACR	LVCF	52.403	9.259	-16.384	5.149	61.456	53.224
500	0.5	4	MAR	LVCF	51.550	9.474	-17.366	5.435	47.334	52.145
500	0.5	8	MACR	LVCF	51.388	4.803	-16.396	2.617	56.704	52.081
500	0.5	8	MAR	LVCF	50.657	4.972	-16.171	2.446	49.819	50.701

Table 5 (continued). Parameter estimates, underlying data subset, and imputation method

500	0.5	4	MACR	Predicted	51.263	9.813	-15.888	5.016	52.304	46.599
500	0.5	4	MAR	Predicted	51.407	9.589	-17.690	5.660	45.494	48.908
500	0.5	8	MACR	Predicted	51.212	4.899	-16.420	2.688	44.745	49.226
500	0.5	8	MAR	Predicted	50.572	5.028	-16.190	2.454	45.736	50.414
500	0.5	4	MACR	Multiple R	51.264	9.802	-15.889	4.997	59.280	45.841
500	0.5	4	MAR	Multiple R	51.274	9.655	-17.584	5.606	46.891	50.177
500	0.5	8	MACR	Multiple R	51.006	4.945	-16.111	2.618	55.544	49.221
500	0.5	8	MAR	Multiple R	50.541	5.035	-16.224	2.477	49.237	50.635
500	0.5	4	MACR	Propensity	51.271	9.762	-15.896	5.064	57.653	46.721
500	0.5	4	MAR	Propensity	51.125	9.740	-17.298	5.447	46.593	50.947
500	0.5	8	MACR	Propensity	50.958	4.947	-16.034	2.621	53.877	51.036
500	0.5	8	MAR	Propensity	50.507	5.047	-16.138	2.448	48.236	51.451

Table 6. Standard errors of Parameter estimates, underlying data subset, and imputation method

SIZE	ICC	TIME	MISSING	Imputation	γ_{00}	γ_{10}	γ_{01}	γ_{11}	σ_0^2	σ_ε^2
500	0.5	4	MACR	Hot-deck	0.782	0.215	1.066	0.293	7.839	7.295
500	0.5	4	MAR	Hot-deck	0.687	0.198	1.004	0.289	6.880	7.221
500	0.5	8	MACR	Hot-deck	0.618	0.073	0.843	0.100	7.530	7.217
500	0.5	8	MAR	Hot-deck	0.550	0.067	0.805	0.098	7.058	7.120
500	0.5	4	MACR	Group mean	0.766	0.219	1.044	0.298	7.255	7.435
500	0.5	4	MAR	Group mean	0.699	0.207	1.021	0.303	6.662	7.548
500	0.5	8	MACR	Group mean	0.608	0.084	0.829	0.115	6.605	8.300
500	0.5	8	MAR	Group mean	0.552	0.073	0.806	0.106	6.720	7.680
500	0.5	4	MACR	LVCF	0.791	0.215	1.109	0.293	7.839	7.295
500	0.5	4	MAR	LVCF	0.687	0.198	1.004	0.289	6.880	7.221
500	0.5	8	MACR	LVCF	0.618	0.073	0.843	0.100	7.530	7.217
500	0.5	8	MAR	LVCF	0.550	0.067	0.805	0.098	7.058	7.120
500	0.5	4	MACR	Predicted	0.727	0.201	0.992	0.274	7.232	6.826
500	0.5	4	MAR	Predicted	0.668	0.192	0.977	0.280	6.745	6.993
500	0.5	8	MACR	Predicted	0.568	0.071	0.775	0.097	6.689	7.016
500	0.5	8	MAR	Predicted	0.536	0.067	0.783	0.098	6.763	7.100
500	0.5	4	MACR	Multiple R	0.745	0.199	1.015	0.272	7.699	6.770
500	0.5	4	MAR	Multiple R	0.669	0.192	0.987	0.281	6.848	7.083
500	0.5	8	MACR	Multiple R	0.608	0.071	0.829	0.097	7.453	7.016
500	0.5	8	MAR	Multiple R	0.548	0.067	0.802	0.098	7.017	7.116
500	0.5	4	MACR	Propensity	0.744	0.201	1.014	0.274	7.593	6.835
500	0.5	4	MAR	Propensity	0.680	0.196	0.994	0.286	6.826	7.138
500	0.5	8	MACR	Propensity	0.606	0.073	0.826	0.099	7.340	7.144
500	0.5	8	MAR	Propensity	0.547	0.068	0.799	0.099	6.945	7.173

6. CONCLUSIONS

The procedures used fort he analysis of multilevel models are unusual in that they allow the same analysis for incomplete responses as for complete responses in longitudinal studies. This feature is not shared by other time series models like AR and MA, which require the same form of missing data analyses, as do general regression models with missing covariates.

In this study, since the analysis for incomplete response is the same as for complete responses there seems to be no reason for restricting analysis to complete cases when multilevel modeling is used for analysis of longitudinal data, and the missingness process can be assumed to be MAR or MCAR. If the incomplete observations result from a non-random missingness process, that is, if the probability of being missing is related to the value which is missing, then both complete cases and full data parameter estimates will be biases, as is true in general for analysis of incomplete data.

Multiple imputation is a valuable technique that allows the use of complete-data statistics on data sets with missing values. Comparing the incomplete and imputation data set analyses results using general linear mixed modeling procedures for the growth data, general linear mixed modeling of incomplete data sets with maximum likelihood method is an effective and flexible way of dealing with missing values.

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