Using Reimputation Methods to Estimate the Variances of Estimates of the American Community Survey Group Quarters Population with the New Group Quarters Imputation Methodology[1]

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1. Introduction

1.1 Problem Statement

The Census Bureau has implemented a new imputation program to enhance ACS estimates of the group quarters (GQ) population for small areas. The GQ imputation presents novel challenges for variance estimation, both because it is a mass imputation with roughly as much imputed data as sampled data, and because the GQ facilities being imputed to are not missing, but rather, not-in-sample.

In previous years, the ACS has implemented successive differences replication (SDR) to estimate variances (Fay and Train, 1995). We understood that naively applying SDR to data augmented by imputation, i.e., treating the imputed records as sample, would lead to a serious underestimation of variances. Hence, starting with the 2011 1-year, 2009-2011 3-year, and 2007-2011 5-year ACS estimates, the ACS program has implemented a method that applies inflation factors to the replicates weights of GQ persons (Asiala and Castro, 2012). While this method is better than the naive variance estimator, it is crude, with the same inflation factor used for all characteristics and for the entire state. A better method would reflect the variances by characteristic and for substate geographies. It was the search for such improvements which was the impetus for exploring reimputation methods.

In this study we assessed the feasibility and soundness of using random groups with reimputation to estimate variances for the 2013 data-year products. We note that ACS GQ data lends itself to the formation of random groups, and thus to reimputation with random groups. We also evaluated the previously used SDR methodology, and the current methodology for estimating variances of the GQ population, SDR with inflation factors. We compared the variances for SDR with inflation factors and random groups with reimputation methods for the 2012 1-year ACS estimates for states and for larger counties. We used standard errors obtained through simulations of the ACS GQ population sampling as a benchmark to compare the alternative methods.

1.2 Overview

The GQ imputation is a random hot deck imputation that copies a whole vector of person characteristics from one person record into the recipient. Whole person imputation preserves the relationships between the characteristics, which yields more realistic imputations and prevents a difficult data editing process.

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[1] This report is released to inform interested parties of ongoing research and to encourage discussion of work in progress. Any views expressed on statistical issues are those of the authors and not necessarily those of the U.S. Census Bureau.

To estimate variances for the 2011 data-year ACS estimates, (2011 1-year, 2009-2011 3-year, and 2007-2011 5-year ACS estimates, which were released mostly in calendar year 2012), and the 2012 data-year ACS estimates, (2012 1-year, 2010-2012 3-year, and 2008-2012 5-year ACS estimates), SDR with inflation factors was used for GQ persons. Imputed person records were initially treated like sample and assigned successive difference replicates (see Asiala and Castro, 2012). Then, an inflation factor was applied to these replicates to account for the imputed data not being sample. To determine these inflation factors, 40 simulations were run that redid the GQ imputation process for each not-in-sample GQ facility. These simulations were analogous to the multiple imputations of multiple imputation methodology. However, they captured only the variation of the random mechanism of the imputation. This is not equivalent to the between-imputation variance as defined for multiple imputation methodology, which captures both the variance of the random mechanism of the imputation and the sampling variation in the imputation. Thus we expected that the method of applying inflation factors lead to some degree of underestimation of the estimates of variance.

Reimputation with replication methods of variance estimation may yield useful estimates of variance in the context of the GQ imputation. If sound, they could provide improved variances estimates for each characteristic and for substate geographies. Reimputation with the jackknife, balanced half-sample and bootstrap replication methods is described in Jun Shao's chapter in the book Survey Nonresponse (Shao, 2002), and reimputation used with the random group method is described in Shao and Tang (2011). The basic method of reimputation is that for each replicate or pseudo-replicate one reimputes for the missing data independently from the restricted donor pool defined by the replicate or pseudo-replicate. Shao's papers show that reimputation methods provide asymptotically unbiased estimates of variance for imputation with deterministic imputations, such as regression imputation, and that the random group and bootstrap methods are unbiased for random imputation such as the random hot deck. While the balanced half-sample and jackknife methods are biased for random imputations, Shao provides adjustments that eliminate these biases asymptotically.

Reimputation methods have some resemblances with multiple imputation that make a brief comparison worth discussing. Variance estimation with multiple imputation and reimputation are similar in computational form, in that both yield for each missing record a set of imputations that is used in the variance estimation. For both methods, these sets of imputations capture both the variation of the imputation's stochastic mechanism, and the variance of imputation attributable to the fact that the imputation is conditioned on data that is itself subject to sampling variation. However, there are important differences. Reimputation methods are strictly a variance estimation methodology, whereas multiple imputation is an imputation methodology, albeit one that yields correct variance estimation in the context of imputation. For a given application, multiple imputation may be superior to other imputation methodologies, or it may be suboptimal; though for other applications it may be impractical or unfeasible to implement. Reimputation is feasible for any sampling and estimation methodology for which one can use replication methods of variance estimation.

2. Background

2.1 The Group Quarters Population

According to the 2010 Census 7,987,323 persons resided in group quarters (GQ) facilities. For ACS estimation we group the specific types of GQ facilities into the following seven major groups.

1) Correctional institutions
2) Juvenile facilities
3) Nursing homes
4) Other long-term care facilities
5) College dorms
6) Military facilities
7) Other non-institutional GQs

2.2 American Community Survey Sampling for the Group Quarters Population

For a better understanding of the issues in this paper, some description of the ACS GQ sample design is needed. The GQ frame is divided into two sampling strata within each state, a small GQ stratum and a large GQ stratum, each with different sampling methods. The small stratum consists of GQ facilities with expected populations of 15 or fewer and GQ facilities closed on April 1, 2010. Small stratum GQ facilities are sampled systematically within each state, sorted by small versus closed on census day, new GQ facility versus previously existing, GQ type (more detailed than the seven major types listed in Section 2.0), and geographical order (county, tract, block, street name, and GQ identifier). The sampling rate varies by state, being higher for states with the smallest GQ populations, but was about 1-in-40 (Marquette, 2011) for many states in the 2008, 2009 and 2010 ACS samples (the GQ sampling rates by state were changed for the 2011 ACS GQ sample). If there are 15 or fewer people found in a small stratum GQ facility, then everyone in the facility is in sample. If there are 16 or more people found in a small stratum GQ facility, then ten people are systematically selected from the facility.

The large stratum includes GQ facilities with expected populations of 16 or more. For each large stratum GQ selected to be in sample, one or more systematic samples of groups of ten people are taken to achieve the state sampling rate. All large GQ facilities in a state are sorted by GQ type and geographical order in the large GQ frame. On the 2007 GQ sampling frame, there were approximately 105,000 small stratum GQ facilities, 77,000 large stratum GQ facilities, and 3,000 facilities with an unknown population which were sampled like the small stratum GQ facilities (U.S. Census Bureau, 2009).

Of salience is that the sampling methodology for the GQ population is designed to produce optimal state-level estimates, as it is only for states or larger geographies that estimates of the characteristics of the GQ population are produced. Only the estimates of the total GQ population are published for geographies smaller than the state. While the sample stratification includes type of GQ and geography, the sampling rates are such that many counties and tracts do not have sample for particular major types of GQ facilities which nevertheless exist within them.

2.3 Overview of the Group Quarters Imputation Methodology

The objective of the GQ imputation is to improve the estimates of the GQ population for counties and tracts, thereby also improving estimates of the total resident population for counties and tracts. The limitations in the sample design can be viewed both in terms of high variances of estimates of the GQ population for substate geographies, as well as in a lack of representation of ACS sample in counties and tracts which are known to have GQ facilities. The description of the methodology given in this section is only an overview, for more details see Asiala et al (2011).

2.3.1 The Basic Approach of the Group Quarters Imputation

The approach to the problem is to populate select GQ facilities without ACS sample with person records copied from in-sample GQ facilities, with appropriate weighting adjustments. This imputation is a whole person imputation and not an item-level imputation. The whole set of person characteristics of the donor is copied to the recipient record (with the exception of geography-dependent variables; see Asiala et al 2011). However, the recipient record maintains the recipient GQ type and current residence geography. Imputing to not-in-sample facilities has the advantage for data processing that the imputed person records function as pseudo-sample and are transparent to the data processing and production of estimates.

2.3.2 The Frame

The listing of GQ facilities to which we potentially impute is the ACS GQ sampling frame. In addition to the GQ listing, an important feature of the frame is population counts, which are needed in determining how many GQ person records to impute to a given GQ. For the GQ imputation the expected population sizes are updated based on observed rates of deleted GQ facilities and population controls.

2.3.3 Identify Group Quarters Facilities that Require Imputation

The GQ imputation imputes persons to a subset of not-in-sample GQ facilities on the frame. The GQ selection procedure gives priority to obtaining representation for each major GQ type group in each county for each year. Hence we refer to it as "county first". Then facilities are selected to establish representation for each major type group at the tract level for every 5-year period. Imputing to all not-in-sample facilities would have required imputing a prohibitively large number of records. The selection of not-in-sample GQ facilities for imputation is prioritized as follows.

- The primary objective is to establish representation of county by major type of GQ in the tabulations for each combination that exists on the frame for each 1-year period.
- A secondary objective is to establish representation of tract by major type of GQ for each combination that exists on the frame, as is reasonably feasible, for each 5-year period.

These priorities lead to a scheme where all large stratum GQ facilities are imputed to, but only a sample of small stratum GQ facilities are imputed to so that the second objective is met. Note that the second objective is relevant only to the 5-year estimates, for which we produce tract-level estimates.

2.3.4 Determining How Many Group Quarters Persons to Impute

How many imputed person records each not-in-sample GQ facility receives is a function of its expected population, which is either modeled or observed from previous years' data. Further, for this determination we make a distinction between small and large GQ facilities. We impute a number of persons into selected not-in-sample GQ facilities to obtain a rate roughly similar to that of the overall sampling rate of the GQ population in a given state. For details see Asiala et al (2011).

2.3.5 Select Donors: The Expanding Search Method

The imputation method is a random hot deck. The donor selection method is referred to as the expanding search approach (Erdman and Nagaraja, 2010). The donor selection procedure chooses from within specific type when the donor to imputation ratio within the specific type is large enough for this to be feasible, and gives preference to donors from facilities that are geographically close. Note that for each year of imputation, donors are selected only from that same year. Once a GQ facility has been selected for imputation, the donor pool for that facility is set to be the first combination of geography and GQ type in the following list in which there is at least one donor per five imputed records needed. Donors are recruited first in the lower ranking step starting with step 1. If a suitable donor is not found in a given step, then the search is repeated in the following step.

1. County and specific type
2. County and major type
3. State and specific type
4. State and major type
5. Division and specific type (a census division is a grouping of states and the District of Columbia; the nine divisions are subdivisions of the four census regions)
6. Division and major type
7. Region and specific type (the four census regions are groupings of states and the District of Columbia)
8. Region and major type
9. Specific type
10. Major type

Note that the weights of the sampled GQ persons do not play a role in the donor selection. Furthermore, there is no grouping of donors, only the search areas.

2.3.6 Weighting with the Group Quarters Imputation

The new imputation methodology implied a new weighting scheme which made a clean break from the old weighting design that was used for ACS estimates released prior to 2012. A key feature of the new weighting procedure is that it is applied to the augmented data, making no distinction between sampled and imputed GQ person records. For details see Asiala, Beaghen, and Navarro (2011).

2.4 Successive Differences Replication

Unbiased estimates of variances for ACS estimates do not exist because of the systematic sample design, as well as the ratio adjustments used in estimation. The SDR was designed to be used with systematic samples for which the sort order of the sample is informative (Fay and Train, 1995), as in the case of the ACS's geographic sort. An advantage of SDR is that the variance estimates can be computed without consideration of the form of the statistics or the complexity of the sampling or weighting procedures, such as those being used by the ACS. Since the start of the ACS, SDR has been the method used to produce estimates of variance of estimates of housing units, households, and persons living in households. It was also the method of estimating the variances of the GQ population through the 2010 ACS. Starting with the implementation of the GQ imputation, SDR has been modified for the GQ population with inflation factors (Asiala and Castro, 2012).

SDR is closely related to the successive differences method of variance estimation. Wolter (1985) describes the successive differences method for estimating variances for systematic samples. Ash (2011) describes in more detail the relationship between SDR and successive differences, showing under what circumstances they are equivalent and how SDR when used by the Census Bureau approximates successive differences.

Applications of SDR were developed to produce estimates of variances for the Current Population Survey (U.S. Census Bureau, 2006), where it has been in use since 1995. SDR was also used for the Census 2000 Long Form estimates (Gbur & Fairchild, 2002). Schindler (2005) compared SDR with the jackknife for Census 2000 sample data and found the SDR compared favorably. Ash (2011) conducted simulations examining empirically SDR with data with several artificial data sets and one real data set with various underlying structures, demonstrating empirically the soundness of the SDR method for certain situations. Wolter (1985) had previously investigated empirically the soundness of the closely related successive differences method.

For producing ACS estimates, the first step in implementing the SDR method is the construction of the replicate factors. Next, replicate base weights (the inverse of the probability of sampling) are calculated by multiplying the base weight for each GQ person (or housing unit) by the factors. The weighting process then is rerun, using each set of replicate base weights in turn, to create final replicate weights. Replicate estimates are created by using the same estimation method as the original estimate, but applying each set of replicate weights instead of the original weights. Finally, the replicate and original estimates are used to compute the variance estimate based on the variability between the replicate estimates and the full sample estimate. For the ACS, the number of replicates is 80. These replicates are translated into 80 replicate weights for each person and housing unit record via a Hadamard matrix. More details of this weighting and processing can be found in U.S. Census Bureau (2009).

2.5 Modification of Successive Differences Replication with Inflation Factors

To implement the GQ imputation, a new variance estimation methodology was needed that would properly account for the imputation variance component in addition to the sampling variance component. Applying SDR naively to the set of sampled and imputed records would result in a significant underestimate of the total variance. For this reason, Asiala and Castro (2012) developed a method to adjust the SDR variance estimate to account for the imputation variance. A key requirement was that any methodology would need to be incorporated into the ACS replicate weight variance methodology in order to minimize the impact on tabulation systems for producing ACS estimates. The ACS is presently limited to a system of variance estimation that is replicate weight based due to our tabulation system. Given that limitation and the available development time, it was decided to employ a relatively simple process of inflating the naive variance estimate using a set of inflation factors.

To determine the inflation factors, Asiala and Castro created a set of 40 imputations. In the context of repeated or multiple imputation, one obtains variance estimates for the within and between imputations (Rubin, 1996). The naive sampling variance is known as the within imputation variance. The between imputation variance is the variance among the resulting estimates stemming from the 40 repeated imputations. From these data, a single set of inflation factors was created that was used to adjust the variances for all characteristics. Due to constraints of the ACS tabulation system, one cannot adjust for the variance due to imputation differently for every characteristic or combination of characteristics. Therefore the goal was to estimate one overall adjustment for the increase in variance due to imputation per state by major type. Asiala and Castro observed that there was considerable variation in the degree of imputation by state and major type and that the amount of sampled data could be relatively thin at lower levels of geography such as county. Thus they created an inflation factor for each characteristic at the state by major type level across counties, and then created an average inflation factor across the characteristics.

The goal for the overall inflation factors was to be able to make one adjustment that would approximate the characteristic-level inflation factors while being slightly conservative to avoid significantly underestimating the variances for some characteristics. For this reason, it was decided that the average inflation factor across characteristics for a given state/major type combination would not be the best choice given that it would tend to underestimate the variances for approximately half of the characteristics. Instead, the overall inflation factor was calculated using two additional considerations. The first was that the average was computed as a weighted average based on the number of data items within the characteristic. The second consideration was combining this weighted average with the maximum characteristic inflation factor by using the minimum of (weighted average + 0.2, maximum inflation factor). This allowed the overall inflation factor to deviate from the weighted average up to smaller of 0.2 or the maximum characteristic inflation factor. The distance of 0.2 covered the difference between the maximum inflation factor and the weighted average approximately 95 percent of the time. The result was that overall inflation factor for each state/type combination was greater than approximately 90 percent of the characteristic inflation factors.

2.6 Variance Estimation with Random Groups with Reimputation

The random group estimator of variance, $V(\hat{\theta})$, is given by (Wolter, 1985)

$$V(\hat{\theta}) = \frac{1}{k(k-1)} \sum_{i=1}^{k} \left(\hat{\theta}_i - \hat{\hat{\theta}}\right)^2$$

Where $\hat{\theta}_i$ are the estimates of θ based on the ith random group, and

$$\hat{\hat{\theta}} = \frac{1}{k} \sum_{i=1}^{k} \hat{\theta}_i$$

Random groups must be formed such that each random group has essentially the same sample design as the parent sample (Wolter, 1985). In the case of the ACS GQ sample, the original sampling design is one-stage stratified sampling with two strata for each state equivalent, and systematic sampling within each stratum. Thus each random group should contain both strata. Wolter discusses the random group method in the context of systematic sampling. The random groups method does not fully reflect variance reduction due to the systematic sampling. With a systematic sampling interval of *t*, with *k* random groups each group would effectively have a systematic sampling interval of *kt*. We conjecture that much of the variance reduction due to the systematic sample is obviated by the GQ imputation. In any case, smaller *k* produces groups more alike regarding the systematic sample.

With reimputation the random group variance estimator $VR(\hat{\theta})$ has the same form as $V(\hat{\theta})$. We denote $\hat{\theta}_i^R$ as the estimates of the parameter based on the ith random group with reimputation. With reimputation the imputation process is conducted independently for each random group, with both the recipient and donor pool determined by each group.

$$VR(\hat{\theta}) = \frac{1}{k(k-1)} \sum_{i=1}^{k} \left(\widehat{\theta_i^R} - \widehat{\widehat{\theta^R}}\right)^2$$

With the random group method of variance estimation a perennial question is how many groups (*k*) to use. The random group variance estimator has smaller bias with fewer but larger groups (Wolter, 1985). However, it is less precise with fewer groups. With the large ACS group sample, bias may be the lesser concern, even with a larger number of groups. Hence larger *k* may prove to have a better mean squared error.

Shao and Tang (2011) show that the method of random groups with reimputation is asymptotically unbiased for simple random sampling without replacement as:

$$n \to \infty,\ n/k \to \infty, \text{ and } n/N \to 0$$

Since *n* and *n/k* are large for the ACS, and *n/N* is not large, bias due to reimputation may not be a concern for our problem. Thus when selecting *k* we can focus on the considerations typical to random groups. That is, the variance-bias tradeoff when selecting *k*, as discussed earlier (Wolter, 1985). Another consideration is to create groups which are like the parent sample. Smaller values of *k*, such as 2, 5, or 10, allow for all certainty GQ facilities to be represented in each group (we discuss why this is so later). Furthermore, smaller *k* better maintains variance gains of systematic sample, and also makes the donor pools for the reimputation larger, and thus more like imputation with the full data set.

2.7 Simulation Research on the Variances of Estimates with the Group Quarters Imputation

In the original research and development of the GQ imputation to obtain estimates of variances for the evaluation of the GQ imputation, Weidman (2011) ran simulations of the ACS sample selection process, simulating 25 runs with Census 2000 100-percent data (the basic demographic questions that all respondents received). For each of the 25 simulations the ACS weighting both with and without the GQ imputation was implemented and estimates produced. The variances estimated through these simulations fully reflect the sampling variation and the variance due to imputation. Weidman (2011) noted reductions in variances in the estimates with GQ imputation compared to the estimates without the GQ imputation.

3. Research Questions and Methodology

3.1 Research Questions

The primary research questions this study attempted to answer were these.

- Did SDR provide sound estimates of the variances of the GQ population for the 2010 1-year ACS (before the GQ imputation was implemented)?
- Were the variance estimates obtained by SDR with inflation factors (the 2012 data-year methodology) sufficiently sound for particular characteristics and for county-level estimates?
- Were the variance estimates obtained by reimputation with random groups sound? How sensitive are the results to the number of random groups used (five or ten)?
- How did the random groups with reimputation variance estimates compare with the SDR with inflation factors by characteristic and for county-level estimates?

We use the adjective sound above in a loose, practical sense. Asiala and Castro (2012) examined the magnitudes of the estimated variances produced by SDR with inflation factors and deemed them acceptable. Ultimately, the assessments made in this report are relative to how the alternatives compare to this current ACS methodology.

3.2 Reimputation Methodology

This section describes the implementation of the reimputation with random groups for the ACS GQ person data with the GQ imputation.

3.2.1 Application of Reimputation Methods to ACS Group Quarters Data

In more typical imputation methods, the units imputed to are sampled units which are nonresponding or have incomplete information, that is, whole unit or item nonresponse. In contrast, in the GQ imputation, the units imputed to are not-in-sample cases. Since this application is unusual, a few words are required to suggest why we think the reimputation may yield useful variance estimates. We can think about variance estimation in terms of repeated sampling along with subsequent imputation and estimation, with the resulting distribution of estimates describing the variance of those estimates. (This is what was done in Weidman's simulations). A variance estimator should obtain the statistics describing the distributions of the estimates under these repeated samplings. While in the production of official ACS estimates we cannot replicate the sampling and imputation in their entirety, the replicate methods try to mimic these processes. We can redo the entire imputation and estimation process within each replicate, which makes it reasonable to believe that replicate methods with reimputation may yield sound variance estimates.

3.2.2 Which Replication Method to Use with Reimputation

Both the random group and delete-a-group jackknife (Kott, 1998) methods could potentially be used with reimputation for the ACS GQ data. Replicate groups can be formed from the ACS GQ data for both. For the random group method we created *k* replicate groups, whereas each of *k* jackknife pseudo-replicates would be formed by deleting one group from the entire data. Furthermore, the random group and delete-a-group jackknife are amenable to the replicate weight framework that is used in the production of ACS estimates and margins of error. In contrast, the ACS GQ data was not sampled in such a manner that it would lend itself to balanced half-sample or bootstrap approaches, nor are these methods amenable to the replicate weight processing which the ACS uses.

In choosing between the random group and delete-a-group jackknife methods with reimputation, the random group has two crucial advantages. First, the random group with reimputation method is unbiased when used with hot deck imputation (Shao, 2002). Second, it required fewer reimputations, just one for each imputed record. In contrast, the jackknife would require *k*-1 reimputations for each imputation, and it is biased with random imputation. (It might be possible to make an adjustment to control this bias with additional multiples of reimputations: Shao, 2002; however, this would have made the jackknife with reimputation even more unwieldy). The greater number of records required and the bias made the jackknife unsuitable for the production of ACS estimates, thus we only evaluated the random group with reimputation in this study.

3.2.3 Implementation of Reimputation with Random Groups

Reimputation with random group replication required forming *k* groups of both donors and recipients. Again, a key feature was that the donors and recipients came from the same group. Details on its implementation follow.

1) When using replicate methods with a systematic sample, the *k* replicate groups were formed by selecting every k^{th} sampled unit to be in the k^{th} group. For the ACS data with GQ imputation, the replicate groups also included not-in-sample units, both those imputed to and those not imputed to during the original, full-sample GQ imputation.
2) Implementing the random group method with reimputation required increasing the number of GQ person records used in the ACS estimation process. Each imputed person record had a reimputed record drawn from its corresponding random group (the group to which the not-in-sample case belongs). Since there were roughly as many imputed GQ person records as interviewed, reimputation with random groups increased the total number of GQ person records in ACS processing by about 50 percent.
3) The reimputed records had zero weight for the production of ACS estimates. However, their replicates weights were not zero and these records contributed to the estimation of the variances.
4) The random groups method can be easily parameterized in the ACS production replicate framework. Group membership is indicated with a replicate factor of one.

Details on the formation of the random groups follow.

5) Because we required each random group to have the same sample design as the parent sample, each certainty GQ facility was selected into each random group. (Large-stratum GQ facilities are selected with probability proportional to size, with the larger facilities being picked with certainty; the certainty cutoff depends on the sampling rate of the state).
6) Each sampled person within a certainty GQ facility was assigned uniquely to one random group. Since persons in large GQ facilities are selected in groups of ten in the ACS sampling, ten was the maximum number of groups we could have while maintaining groups with the same number of respondents within the certainty facilities. The number of groups had to be a factor of 10, i.e., 2, 5, or 10.
7) The noncertainty large-stratum GQ facilities, both sampled and not-in-sample, were assigned to random groups with a take-every of *k*. For the sampled noncertainty large stratum GQ facilities, all ten persons were assigned to the same random group.
8) The small-stratum GQ facilities, both sampled and not-in-sample, were selected to be in a random group with a take-every of *k*. For the sampled small-stratum GQ facilities, all sampled persons were assigned to the same random group.
9) For the small-stratum and noncertainty large-stratum GQ facilities, we maintained the GQ sampling sort order before selection and select both sampled and not-in-sample units in this order to form the *k* groups. If the take-every was *t*, their assignment into *k* random groups effectively increased the take every to *kt*.

4. Metrics

4.1 Sets of Variance Estimates

The research involved comparisons between several sets of variance estimates, which are listed below. While we expected the estimates of characteristics to differ among the 2000-based simulations, 2010 1-year ACS estimates, and 2012 1-year ACS estimates, this was not a limitation. The variances were nonetheless comparable, as the sample design for GQ persons was essentially the same for the various sets of estimates.

- The methodology for the production of 2012 1-year ACS estimates, which applied inflation factors to replicate weights.
- Reimputation with random groups of five and ten replicate groups for the 2012 1-year estimates.
- Methodology for the production of 2010 1-year ACS estimates, the last data-year before the implementation of the GQ imputation.
- Variances estimates based on 25 simulations of Census 2000 data (Weidman, 2011), both with and without the GQ imputation.

4.2 Research Characteristics

We included in the study the person characteristic data listed below. We examined all major demographics collected in the decennial census, but just a select few characteristics collected only in the ACS. The Census 2000-based simulations included only those demographic characteristics collected for every person, i.e., the 100-percent data.

Demographic Characteristics Collected in the Census 2000 (100-percent Data)

- Age groups: under 18, 18-34, and 65+
- Hispanic origin
- Race groups (alone): white, black, and Asian/Pacific islander
- Sex

Characteristics not collected in the Census 2000 (non 100-percent Data)

- Marital status
- Educational attainment (population 3 years and older)
- Speaks a language other than English at home (population 5 years and older)
- Disability status (civilian population 3 years and older)

We chose to examine characteristics whose estimates were in the form of proportions or totals because their calculations were straightforward. Some other characteristics, such as poverty or median income, would bring with them complexities of calculation that would not further the aims of the research. Further, we examined the estimates of variances of proportions rather than totals because the scaling makes the variances of various characteristics more comparable. By choosing to examine proportions we lost little generality, as the coefficients of variation for estimates of totals are equal to the coefficients of variation for estimates of proportions when the denominators in the proportions are controlled to, as is the case for many of the ACS demographics.

4.3 Summary Statistics

The summary statistics included mean and root mean square (RMS) differences between standard errors produced by the alternative variance estimators. We made these summaries over the 51 1-year ACS estimates for the state level variances of characteristics, and over groupings of 185 and 213 1-year ACS estimates of larger counties, depending on the size of the GQ population.

4.4 Criteria for Evaluation

We used the Census 2000-based simulations as a benchmark or truth deck. Since they are based on 25 random realizations of the sampling, these estimates of variances themselves are subject to sampling error. However, the 2010 and 2012 1-year ACS estimates were based on only one realization of the sampling process. Thus the sampling error of the variance estimates based on simulations is small compared to the sampling error of variance estimates based on the 2010 or 2012 data, perhaps on the order of $1/25^{th}$. In addition to examining the 2010 1-year ACS estimates (produced without the GQ imputation) for the soundness of the SDR, these estimates served as a useful upper bound for variance estimates, since we know the GQ imputation reduced the variances of the estimates (Weidman, 2011). The summary statistics described in Section 4.4 were the criteria for assessing the alternative variance estimators. However, we also plotted graphs of the distribution of SEs for the 51 states for two methods being compared.

5. Results

In the Appendix we show the estimated SEs for the proportion of males in each state equivalent for the seven different approaches listed below. Similar tables for the other characteristics studied in this report can be found in the full, final Census Bureau report for this project (Beaghen, 2014). The tables and graphs found in this chapter are based on these and similar calculations for other characteristics.

Comparison data sets found in the Appendix.

1) Census-2000 based simulations with GQ imputation; this was the baseline for many of our comparisons.
2) SDR with inflation factors (research approach) for 2012 1-year ACS
3) SDR with inflation factors (as used in the production of ACS estimates) for 2012 1-year ACS
4) Ten Random groups with reimputation for 2012 1-year ACS
5) Five Random groups with reimputation for 2012 1-year ACS
6) Census-2000 based simulations with no GQ imputation
7) 2010 1-year ACS (no GQ imputation)

5.1 Evaluating the Successive Differences Replication by Comparing to the Simulations

We first considered the SDR without the GQ imputation, comparing its results to the 25 simulations. We did this for five of the characteristics included in the simulations. Table 1 shows the correlations over the 51 state equivalents of their SEs calculated by the Census 2000-based simulations (with no GQ imputation) and their SEs calculated by the SDR for the 2010 1-year ACS estimates. For example, the correlation of the two sets of SEs of the proportion male is 0.643.

Table 1: Correlation of the Standards Errors of Proportions of Characteristics over States between Simulations and Successive Differences Replication for ACS 2010 1-Year Estimates

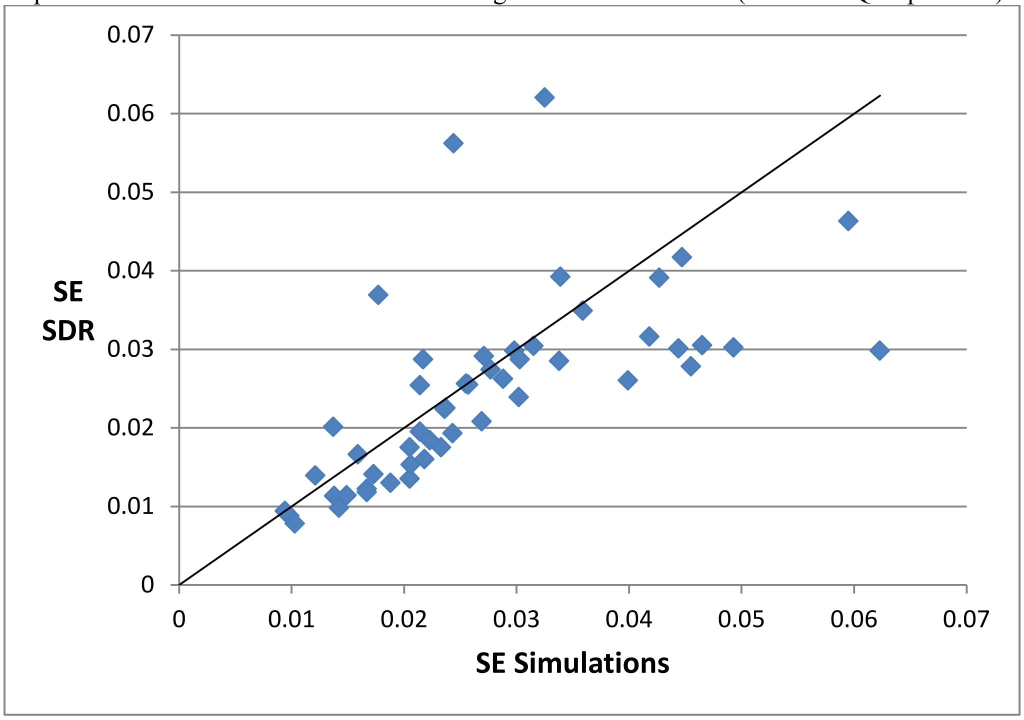
Characteristic	Correlation
Male	0.643
Hispanic Origin	0.344
Age: 65 Years or Older	0.623
Age: 18-34 Years	0.771
Age: 17 Years or Younger	0.392

Source: 2010 American Community Survey 1-year Data and Simulated Data

The data used to produce the correlations in Table 1 are plotted for the characteristic male in Figure 1. In the graph in Figure 1 each of the 51 points corresponds to one state equivalent. Note that the larger states have larger GQ populations and thus larger samples and smaller SEs. As one looks at states with higher SEs one finds smaller populations. For example, the point (0.0325, 0.062) on the X-axis and Y-axis corresponds to Montana, and the point (0.0595, 0.0463) corresponds to Utah.

For two characteristics, Hispanic origin, and 17 years of age or younger, the correlations were less than 0.5. However, their means are about the same (Beaghen, 2014). The similarities of the estimates of SEs between the SDR and the simulation generally support the soundness of the SDR approach for estimating the variances of estimates of the GQ population.

Figure 1: Scatterplot of Standard Errors of Proportion Male in GQ Population for States: Successive Differences Replication for 2010 1-Year ACS Estimates against the Simulations (with no GQ Imputation)



Source: 2010 American Community Survey 1-year Data and Simulated Data

5.2 Statistically Significant Differences between 2011 and 2012 1-Year ACS Estimates of Characteristics in the Group Quarters Population

Before examining SEs in the context of the GQ imputation we first note some supporting results from preliminary work. Jordan (2013) noted a high number of statistically significant differences when comparing the state-level estimates of multiple characteristics in the GQ population from 2011 to 2012. His observation was consistent with our expectation that the current ACS methodology for estimating the variances of estimates of characteristics of the GQ population leads to underestimates. For perspective, he also compared the state-level estimates of the same characteristics from 2008 to 2009. He compared multiple demographic and nondemographic characteristics for a total of 2,846 tests and 2,861 tests (51 state equivalents by 58 characteristic levels; though not all states had data for all characteristic levels). Between 2011 and 2012 24.2% of the differences were statistically significant at the 10% level. If there were no real change from year to year, and the variances used in the hypothesis tests were sound, then we would expect about 10% of the tests to be statistically significant. Because there are some genuine changes from year to year, Jordan looked at the 2008 to 2009 ACS GQ population. He found that there were 17.9% statistically significant changes. These results are consistent with an underestimation of the estimation of variances, though more genuine changes from 2011 to 2012 cannot be ruled out.

5.3 State-Level Analysis Evaluating the Successive Differences Replication by Comparing to the Simulations

Next we consider variance estimation in the context of the GQ imputation for state-level estimates. This analysis allows us to investigate the SEs by characteristic. It also allows us to gain insight into any degree of underestimation in the SEs yielded by the current methodology used to produce ACS estimates.

The mean differences, calculated over 51 state equivalents, as compared with the SE estimated by the simulation indicate possible under- or overestimation of the SE for a given method. In Table 2 we see these mean differences. The comparisons of the SDR with inflation factors to the simulations confirm a moderate underestimation of variance. For both the SDR with inflation factors, both the research and the production approach, the SEs were smaller than those of the simulations (in Table 2 we see that as a negative sign). For example, for the proportion male the SE of the SDR with inflation factors (production approach) averaged 0.006759 less than that of the SE from the simulations; (for comparison, the median SE for male by the simulations was 0.0189).

Table 2: Mean Difference in Estimates of SEs between Four Alternative Variance Estimators and the SEs Estimated by Simulations

Characteristic	Ten Random Groups with Reimputation	Five Random Groups with Reimputation	SDR with Inflation Factors - Research Approach	SDR with Inflation Factors - Production Approach
Male	-0.003441	-0.004467	-0.006001	-0.006759
Hispanic Origin	0.001554	0.000076	-0.000209	-0.001061
Age 65 years and Over	-0.000002	0.001002	0.001049	-0.000513
Age Under 18 years	-0.002194	-0.001572	-0.002459	-0.003027
Age 18 to 34 years	0.000590	0.000682	0.000006	-0.001279

Source: 2012 American Community Survey 1-year Data and Simulated Data

The root mean square (RMS) difference, calculated over 51 state equivalents, between an alternative estimate of SE and the simulation SE, is a measure of how close the alternative SE is to simulation SE. Again, if we consider the simulation SE as our benchmark, the RMS difference serves as a tool to evaluate the alternative SEs. In Table 3 we see the RSM differences between the alternative SEs and the simulation SE over five characteristics. We note that the SDR with inflation factors research approach is modestly closer to the simulations than the production approach. We expected this result as the inflation factors were capped at 1.4, though the research approach suggested higher caps for several state/GQ type combinations. Clearly, the ten random groups with reimputation outperformed the five, with consistently smaller RMS differences. However, we see no advantage for the random group with reimputation methods over the SDR with inflation factors approaches; the RMS differences for the SDR with inflation factors production approach were consistently as small as or smaller than those of the reimputation.

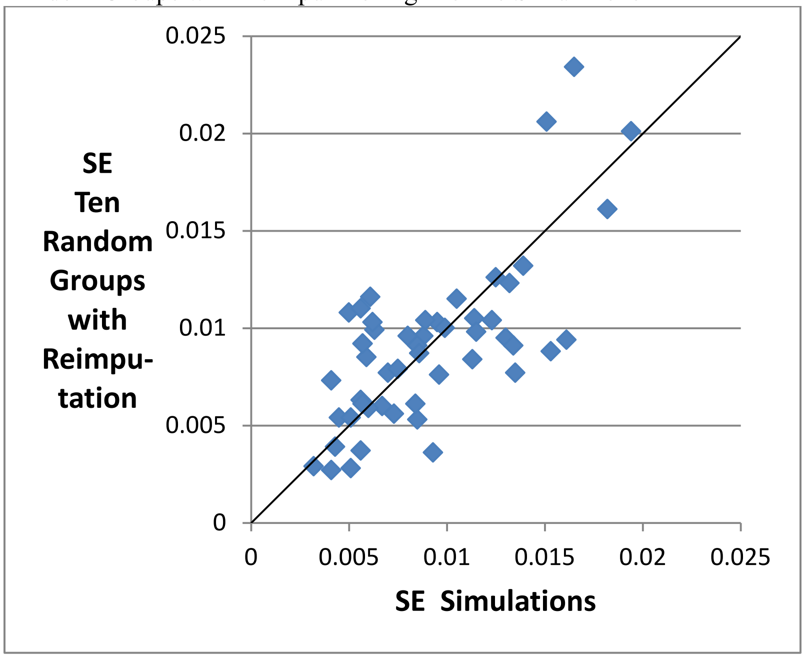
Table 3: Root Mean Square Differences of SEs of Proportions Calculated by Four Alternatives with the SEs Estimated by Simulations

Characteristic	Ten Random Groups with Reimputation	Five Random Groups with Reimputation	SDR with Inflation Factors - Research Approach	SDR with Inflation Factors - Production Approach
Male	0.001174	0.001490	0.001267	0.001346
Hispanic Origin	0.000453	0.000373	0.000265	0.000287
Age 65 years and Over	0.000435	0.000753	0.000407	0.000352
Age Under 18 years	0.000596	0.000658	0.000576	0.000623
Age 18 to 34 years	0.000703	0.000930	0.000485	0.000506

Source: 2012 American Community Survey 1-year Data and Simulated Data

In Figure 2 we plot the SEs of the estimates of proportion age 65 years or older for the 51 state equivalents for the ten random groups with reimputation against the simulations. From Table 4 we note that the correlation is 0.713. The points generally follow the indicating equal values (shown on the plot), suggesting the soundness of the method of ten random groups with reimputation. As in the graph in Figure 1, in Figure 2, the larger states are found in the lower left hand part of the graph, while the smaller states are found in the upper right hand part.

Figure 2: Scatterplot of Standard Errors of Proportion Age 65 Years or Older in GQ Population for States: Ten Random Groups with Reimputation against the Simulations



Source: 2012 American Community Survey 1-year Data and Simulated Data

Table 4: Correlation between the Standards Errors for States of Ten Random Groups with Reimputation and Simulations (with GQ Imputation)

Characteristic	Correlation
Male	0.657
Hispanic Origin	0.820
Age: 65 Years or Older	0.713
Age: 18-34 Years	0.754
Age: 17 Years or Younger	0.336

Source: 2012 American Community Survey 1-year Data and Simulated Data

In Tables 5 and 6 we see comparisons of SEs by the alternative methods for characteristics that were not on the Census 2000. Hence, we could not compare them to the Census 2000 simulations. Instead, we made three comparisons; the ten random groups and the five random groups were each compared to the SDR with inflation factors research approach; in addition, we compared the ten random groups to the SDR with inflation factors production approach. In Table 5 we see the random group estimates have larger estimates of SEs than the production research, as reflected by values greater than zero. This is consistent with the SDR with inflation factors underestimating the SEs. In Table 6 we see that the SEs for the ten random groups were consistently closer to the production research than the five random groups. While we did not have a benchmark for these characteristics, such as the simulations, it is evidence for the superiority of ten random groups over five. Again, we see no evidence that the SDR with inflation factors approaches performed poorly by individual characteristic.

Table 5: Mean Difference of Estimates of SEs over States - Three Comparisons

Characteristic	Ten Random Groups versus SDR with Inflation Factors Research Approach	Five Random Groups versus SDR with Inflation Factors Research Approach	Ten Random Groups versus SDR with Inflation Factors Production Approach
High School Degree or Better	-0.001209	-0.001526	-0.001482
Married	0.002256	0.000719	0.002486
Race: White Alone	0.002568	0.002624	0.002500
Race: Black Alone	0.001680	0.001515	0.001652
Race: Asian Alone	0.001944	0.001339	0.001982
Speaks Another Language at Home and Speaks English Less than Very Well	-0.001042	-0.001331	-0.001099
With a Disability	0.000435	-0.000870	0.000267

Source: 2012 American Community Survey 1-year Data and Simulated Data

Table 6: Root Mean Squared Differences of Estimates of SEs over States - Three Comparisons

Characteristic	Ten Random Groups versus SDR with Inflation Factors Research Approach	Five Random Groups versus SDR with Inflation Factors Research Approach	Ten Random Groups versus SDR with Inflation Factors Production Approach
High School Degree or Better	0.000758	0.000865	0.000806
Married	0.000608	0.000478	0.000650
Race: White Alone	0.000783	0.000769	0.000786
Race: Black Alone	0.000563	0.000617	0.000563
Race: Asian Alone	0.000410	0.000377	0.000411
Speaks Another Language at Home and Speaks English Less than Very Well	0.000440	0.000516	0.000444
With a Disability	0.000669	0.000795	0.000689

Source: 2012 American Community Survey 1-year Data and Simulated Data

5.4 County-Level Analysis

Because the method of applying inflation factors to SDR was designed to be optimal at the state level, there was concern that they would perform worse for substate geographies. This might be particularly likely if the degree of imputation was uneven across substate geographies. To investigate this possibility we examined comparisons for subsets of larger counties, that is, those with 65,000 or greater population, for which ACS estimates are published annually. We present two separate analyses: one for counties with total GQ populations of 10,000 or more (there were 185 such counties), shown in Table 7; and one for counties with total GQ populations of 5,000 to 9,999 (there were 213 such counties), shown in Table 8. We also looked at counties with GQ populations of 2,000 to 4,999 and saw similar patterns, though for economy we did not include those results in this report (Beaghen, 2014).

Table 7: Counties with 10,000 or more GQ Population: Comparing SEs of Proportions by Two Alternative Methods to SEs of the Simulations

Characteristic	Mean Difference Ten Random Groups and Simulations	Mean Difference SDR with Inflation Factors Production Approach and Simulations	RMS Difference Ten Random Groups and Simulations	RMS Difference SDR with Inflation Factors Production Approach and Simulations
Male	-0.023531	-0.021252	0.002528	0.002156
Hispanic Origin	0.000446	-0.002508	0.000851	0.000648
Age 65 years and Over	-0.001290	-0.000477	0.001118	0.000902
Age Under 18 years	-0.005315	-0.004596	0.001011	0.000915
Age 18 to 34 years	-0.008970	-0.011308	0.001468	0.001446

Source: 2010 American Community Survey 1-year Data and Simulated Data

In both Table 7 and Table 8 we see the RMS differences in SEs between the random groups and simulations, and between the SDR with inflation factors and the simulations. These sets of differences are similar, though the SDR with inflation factor SEs are perhaps a bit closer to the simulation SEs than the random groups SEs are. On the other hand, we continue to see the tendency for the SDR with inflation factor SEs to be smaller than that exhibited by the simulations; the mean differences between the SDR with inflation factor SEs and those of the simulations are generally negative.

Table 8: Counties with 5,000 to 9,999 GQ Population: Comparing SEs of Proportions by Two Alternative Methods to the SEs of the Simulations

Characteristic	Mean Difference Ten Random Groups and Simulations	Mean Difference SDR with Inflation Factors Production Approach and Simulations	RMS Difference Ten Random Groups and Simulations	RMS Difference SDR with Inflation Factors Production Approach and Simulations
Male	-0.041298	-0.033397	0.003913	0.003532
Hispanic Origin	-0.000677	-0.003169	0.001257	0.001068
Age 65 years and Over	0.004058	0.003612	0.002292	0.001983
Age Under 18 years	-0.003497	-0.002163	0.000925	0.000850
Age 18 to 34 years	-0.014419	-0.019614	0.002265	0.002581

Source: 2012 American Community Survey 1-year Data and Simulated Data

6. Conclusions

Our most general conclusion was that while random groups with reimputation could be successfully implemented in the ACS and yielded plausible variance estimates, it was not shown to be superior to the current ACS methodology as applied to ACS data. More detailed conclusions follow.

1. The successive differences replication (SDR), used for the GQ estimates through data-year 2010 and currently in use for the household population, yielded estimates of SE that were comparable to those of the simulations. This supports the soundness of the SDR approach for the estimates of variances of GQ population without the GQ imputation.
2. The production approach, SDR with inflation factors, appeared to moderately underestimate the SEs of estimates, a result that was expected. That said, it yielded results at least as close to the simulations as the random groups with reimputation.
3. The SDR with inflation factors, the production approach, yielded SEs for state-level estimates similar too but slightly lower than that of the research approach. The maximum value of 1.4 for the inflation factor was responsible for this difference.
4. At the county level we did not see a degradation of reliability of the variance estimates of the SDR with inflation factors. It performed comparably to those of the reimputation with random groups when compared to the simulated results. This observation dispels concerns that, since the inflation factors were optimized at the state level, that they would not work well at the county level.

5. The random groups with reimputation method was successfully implemented and produced comparable results for estimating the variances of the GQ population in conjunction with the mass GQ imputation and a complex weighting methodology. While the theory of reimputation with random groups has been laid out and the method demonstrated in other scenarios, it was not clear how it would work with a weighting methodology as complex as that of the ACS. In addition to the controlling to state-level totals (by major GQ type), there are tract-level constraints that smooth the estimates. Random groups with reimputation revealed itself as robust to these weighting complexities and to a high degree of imputation.
6. Five random groups reimputation had noticeably more unreliability than did the ten groups, an expected result.
7. Importantly, this study did not estimate the sampling error of the estimated SEs themselves. Some sense of the sampling error can be gained by looking at the graphical comparison of the alternative methods against those of the simulations, since those of the simulations are more reliable. The graphical displays do not provide any reason to doubt the general conclusions made, though a more rigorous examination of the sampling errors would be valuable.
8. There is no evidence that the production approach to variance estimation with GQ imputation, SDR with inflation factors, needs to be revised immediately.

7. Future Research

The results of this work point to some worthwhile additional investigation.

1. Test the production methodology with higher values of the inflation factors. A straightforward alternative would be to increase each inflation factor by 0.1, up to the limit of 1.4. We might expect SEs slightly closer to those of the simulations.
2. Obtain the simulation results for the major race groups white and black, and include these in the comparisons. (The documentation of the race coding of the Census 2000-based simulations was not available in time for this report).
3. Attempt the random groups with reimputation with 20 groups to achieve more reliable estimates. Creating 20 groups would require violating one of the rules for that construction of random groups; namely, that each sampled person appear in only one group. In particular, persons in GQ facilities which were sampled once but were eligible to be sampled twice would have to be assigned to two random groups. For example, if the sampling rate for the state were 1/40, persons in GQ facilities with expected populations of 401 to 799 would have to be assigned to two groups, even if just one GQ was actually selected in sample. Including some GQ persons in two random groups would lead to an underestimate of the variance. First, two groups with the same persons would not differ to fully reflect the sampling variation; and second, the effective sample size would appear larger than it is. However, if the number of such GQ facilities is not too large these may be tolerable limitations, and we may obtain more reliable estimates of SE via the random groups with reimputation.
4. In addition to forming random groups that maintain the sequential ordering of the sample selection, we could form random groups where group membership is selected at random. We could repeatedly sample in this manner to form multiple sets of random groups. With these simulated random groups we could create a distribution of variance estimates to evaluate their sampling error or reliability.
5. A second way to assess the sampling error of the estimates of the SEs would be to exploit the regression error between the alternative variances estimators and the simulations. If we assumed the SEs as estimated by the simulations were truth, then the regression error would yield an estimate of the sampling error of the SEs. This approach would be limited by the fact that the SEs as determined by the simulations themselves have sampling error. Thus it would be only a crude approach, but it may be sufficient to better understand the limitations of the variance estimation methods.

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APPENDIX: Standard Errors of Estimates of the Proportion Male in the GQ population for Alternative Methods

Standard Errors of the Proportion Male in the GQ Population							
State	Simulations with GQ Imputation	SDR with Inflation Factors Research	SDR with Inflation Factors Production	Ten Random Groups with Reimputation	Five Random Groups with Reimputation	Simulations No GQ Imputation	SDR No GQ Imputation
AL	0.0195	0.0124	0.0123	0.0065	0.0083	0.0255	0.0256
AK	0.0216	0.0209	0.0191	0.0127	0.0166	0.0298	0.0298
AZ	0.0188	0.0108	0.0101	0.0112	0.0204	0.0243	0.0193
AR	0.0204	0.0138	0.0137	0.0167	0.0163	0.0427	0.0391
CA	0.0070	0.0045	0.0040	0.0064	0.0041	0.0098	0.0088
CO	0.0153	0.0109	0.0102	0.0146	0.0200	0.0217	0.0287
CT	0.0176	0.0107	0.0099	0.0191	0.0098	0.0205	0.0135
DE	0.0240	0.0232	0.0204	0.0264	0.0156	0.0315	0.0304
DC	0.0251	0.0215	0.0225	0.0204	0.0346	0.0359	0.0349
FL	0.0061	0.0057	0.0053	0.0085	0.0031	0.0138	0.0113
GA	0.0102	0.0077	0.0072	0.0166	0.0145	0.0159	0.0166
HI	0.0222	0.0210	0.0201	0.0197	0.0218	0.0277	0.0274
ID	0.0278	0.0246	0.0251	0.0239	0.0254	0.0447	0.0417
IL	0.0118	0.0071	0.0068	0.0093	0.0144	0.0167	0.0122
IN	0.0159	0.0097	0.0095	0.0123	0.0145	0.0205	0.0175
IA	0.0186	0.0128	0.0115	0.0110	0.0077	0.0237	0.0225
KS	0.0190	0.0128	0.0113	0.0131	0.0086	0.0288	0.0262
KY	0.0189	0.0104	0.0097	0.0160	0.0165	0.0257	0.0255
LA	0.0130	0.0115	0.0113	0.0129	0.0086	0.0177	0.0369
ME	0.0322	0.0206	0.0197	0.0366	0.0326	0.0444	0.0301
MD	0.0117	0.0103	0.0091	0.0161	0.0042	0.0223	0.0184
MA	0.0143	0.0082	0.0076	0.0181	0.0143	0.0188	0.0130
MI	0.0126	0.0078	0.0071	0.0125	0.0086	0.0149	0.0114
MN	0.0204	0.0129	0.0126	0.0196	0.0179	0.0233	0.0175
MS	0.0197	0.0134	0.0132	0.0151	0.0125	0.0271	0.0291
MO	0.0127	0.0105	0.0100	0.0172	0.0167	0.0206	0.0153
MT	0.0223	0.0234	0.0227	0.0181	0.0291	0.0325	0.0620
NE	0.0398	0.0173	0.0156	0.0234	0.0157	0.0493	0.0302
NV	0.0139	0.0201	0.0204	0.0210	0.0209	0.0244	0.0562
NH	0.0234	0.0218	0.0183	0.0307	0.0122	0.0339	0.0392
NJ	0.0122	0.0077	0.0072	0.0057	0.0112	0.0167	0.0118
NM	0.0261	0.0263	0.0254	0.0182	0.0252	0.0338	0.0285
NY	0.0073	0.0052	0.0047	0.0061	0.0075	0.0103	0.0078
NC	0.0120	0.0070	0.0066	0.0103	0.0078	0.0121	0.0139
ND	0.0506	0.0243	0.0234	0.0156	0.0088	0.0623	0.0298
OH	0.0083	0.0070	0.0064	0.0088	0.0128	0.0137	0.0201
OK	0.0178	0.0123	0.0122	0.0155	0.0171	0.0214	0.0254
OR	0.0204	0.0137	0.0124	0.0163	0.0233	0.0302	0.0239
PA	0.0092	0.0058	0.0051	0.0038	0.0036	0.0142	0.0098
RI	0.0316	0.0197	0.0177	0.0384	0.0402	0.0465	0.0305
SC	0.0218	0.0109	0.0112	0.0093	0.0042	0.0269	0.0208
SD	0.0298	0.0211	0.0195	0.0286	0.0352	0.0455	0.0278
TN	0.0190	0.0108	0.0108	0.0067	0.0034	0.0236	0.0224
TX	0.0076	0.0053	0.0052	0.0037	0.0031	0.0094	0.0094
UT	0.0528	0.0171	0.0166	0.0277	0.0190	0.0595	0.0463
VT	0.0243	0.0232	0.0210	0.0178	0.0233	0.0399	0.0260
VA	0.0098	0.0081	0.0078	0.0126	0.0132	0.0173	0.0141
WA	0.0171	0.0099	0.0090	0.0157	0.0088	0.0214	0.0195
WV	0.0347	0.0194	0.0190	0.0264	0.0179	0.0418	0.0316
WI	0.0163	0.0102	0.0095	0.0111	0.0156	0.0218	0.0160
WY	0.0310	0.0265	0.0238	0.0363	0.0210	0.0303	0.0287

Source: 2010 and 2012 American Community Survey 1-year Data and Simulated Data