Fitting a Bayesian Fay-Herriot Model

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Washington, DC
October 25, 2018

Disclaimer

The Findings and Conclusions in This Preliminary Presentation Have Not Been Formally Disseminated by the U.S. Department of Agriculture and Should Not Be Construed to Represent Any Agency Determination or Policy.

Overview

- NASS interest in small area estimation (SAE)
- The Fay and Herriot (1979) model
- Case study: county estimates of planted corn, Illinois 2014
  - Computation in R and JAGS

Small Area Estimation (SAE) Literature

"A domain is regarded as 'small' if the domain-specific sample is not large enough to support [survey] estimates of adequate precision."–Rao and Molina (2015)

Regression and mixed-modeling approaches in SAE literature

- Shrinkage–improve estimates with other information
- Utility of auxiliary data as covariate
- Variance-bias trade off

Two common models

1. Unit-level models, e.g., Battese et al. (1988)
   - USDA NASS (formerly SRS) as source of data/funding
2. Area-level models, e.g., Fay and Herriot (1979)

NASS Interest In SAE

Iwig (1996): USDA's involvement in county estimates in 1917

Published estimates used by:

- Agricultural sector
- Financial institutions
- Research institutions
- Government and USDA

Published estimates used for:

- County loan rates
- Crop insurance
- County-level revenue guarantee

National Academies of Sciences, Engineering, and Medicine (2017)

- Consensus estimates: Board review of survey and other data
- Currently published without measures of uncertainty
- Recommends transition to system of model-based estimates

Fay-Herriot (Area-Level) Model

Fay and Herriot (1979)–improved upon per capita income estimates with following model

$$\begin{aligned} \hat{\theta}_j &= \theta_j + e_j, \quad j = 1, \ldots, m \textit{ counties} \qquad (1) \\ \theta_j &= \boldsymbol{x}_j' \boldsymbol{\beta} + u_j \qquad (2) \end{aligned}$$

Adding Eqs. 1 and 2

$$\hat{\theta}_j = \boldsymbol{x}_j' \boldsymbol{\beta} + u_j + e_j$$

- $\hat{\theta}_j$, direct estimate
- $E(e_j|\theta_j) = 0$
- $V(e_j|\theta_j) = \hat{\sigma}_j^2$, estimated variance
- $\boldsymbol{x}_j$, known covariates
- u_j, area random effect
- $u_j \overset{iid}{\sim} (0, \sigma_u^2)$

Fay-Herriot Formulated As Bayesian Hierarchical Model

'Recipe' for hierarchical Bayesian model as in Cressie and Wikle (2011)

Data model:

$$\hat{\theta}_j | \theta_j, \boldsymbol{\beta} \stackrel{ind}{\sim} N(\theta_j, \hat{\sigma}_j^2) \tag{3}$$

Process model:

$$\theta_j | \boldsymbol{\beta}, \sigma_u^2 \stackrel{iid}{\sim} N(\boldsymbol{x}_j' \boldsymbol{\beta}, \sigma_u^2) \tag{4}$$

Prior distributions on $\boldsymbol{\beta}$ and σ_u^2

- Browne and Draper (2006), Gelman (2006): $\sigma_u^2 \sim ?$
- We will specify $\sigma_u^2 \sim Unif(0, 10^8)$, $\boldsymbol{\beta} \stackrel{iid}{\sim} MVN(\mathbf{0}, 10^6 \boldsymbol{I})$

Goal: Obtain posterior summaries about county totals, θ_j

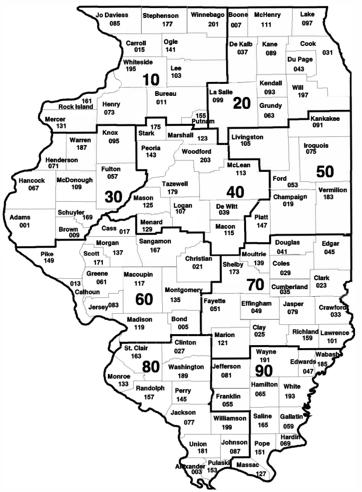
County Agricultural Production Survey (CAPS)

Case study in Cruze et al. (2016)

Illinois planted corn

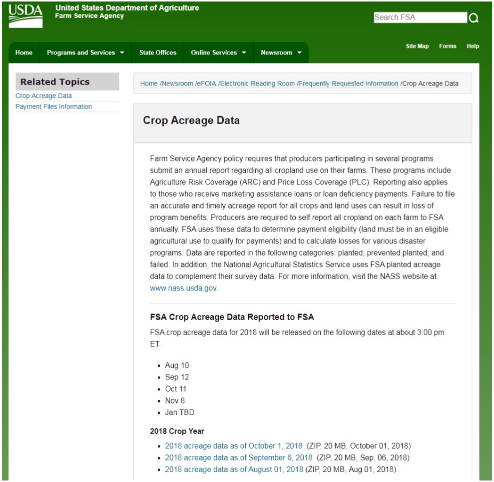
- 9 Ag. Statistics Districts
- 102 counties
- a major producer of corn
- End-of-season survey
  - Direct estimates of totals
  - Estimated sampling variances

	Min	Median	Max
n reports	2	47	93
CV (%)	9.1	19.2	92.3



https://www.nass.usda.gov/Charts_and_Maps/Crops_County/indexpdf.php

Covariate x_1: USDA Farm Service Agency (FSA) Acreage


USDA United States Department of Agriculture Farm Service Agency

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Crop Acreage Data

Farm Service Agency policy requires that producers participating in several programs submit an annual report regarding all cropland use on their farms. These programs include Agriculture Risk Coverage (ARC) and Price Loss Coverage (PLC). Reporting also applies to those who receive marketing assistance loans or loan deficiency payments. Failure to file an accurate and timely acreage report for all crops and land uses can result in loss of program benefits. Producers are required to self report all cropland on each farm to FSA annually. FSA uses these data to determine payment eligibility (land must be in an eligible agricultural use to qualify for payments) and to calculate losses for various disaster programs. Data are reported in the following categories: planted; prevented planted; and failed. In addition, the National Agricultural Statistics Service uses FSA planted acreage data to complement their survey data. For more information, visit the NASS website at www.nass.usda.gov.

FSA Crop Acreage Data Reported to FSA

FSA crop acreage data for 2018 will be released on the following dates at about 3:00 pm ET.

- Aug 10
- Sep 12
- Oct 11
- Nov 8
- Jan TBD

2018 Crop Year

- 2018 acreage data as of October 1, 2018 (ZIP, 20 MB, October 01, 2018)
- 2018 acreage data as of September 6, 2018 (ZIP, 20 MB, Sep. 06, 2018)
- 2018 acreage data as of August 01, 2018 (ZIP, 20 MB, Aug 01, 2018)

https://www.fsa.usda.gov/news-room/efoia/electronic-reading-room/frequently-requested-information/crop-acreage-data/index

- FSA administers farm support programs
- Enrollment popular, not compulsory
- Data self-reported at FSA office
- Administrative vs. physical county

Covariate x_2: NOAA Climate Division March Precipitation

Weather as auxiliary variable

- March: Planting 'intentions'
- April: Illinois planting
- **Could rainfall in March affect planting?**
- One-to-one mapping: ASD and climate division
- Repeat value for all counties within ASD

ASD	Precip (in)
10	1.08
20	1.35
30	1.27
40	1.66
50	1.50
60	1.36
70	1.46
80	1.69
90	2.00

Source: `ftp://ftp.ncdc.noaa.gov/pub/data/cirs/climdiv`
Details in Vose et al. (2014)

NASS Official Statistics

From prior publication: Illinois 2014, 11.9 million acres of corn planted

- Require: State-ASD-county benchmarking of estimates

USDA United States Department of Agriculture
National Agricultural Statistics Service

Quick Stats

Home | Recent Statistics | Developers | Help

Navigation History: Data

Double click any cell below to filter the data by that item. Right click on column heading to pivot or hide columns.

Save :: Spreadsheet :: Printable :: Map :: (10 rows)

Program	Year	Period	Geo Level	State	State ANSI	Ag District	Ag District Code	Data Item	Domain	Value
SURVEY	2014	YEAR	STATE	ILLINOIS	17	...	...	CORN - ACRES PLANTED	TOTAL	11,900,000
SURVEY	2014	YEAR	AGRICULTURAL DISTRICT	ILLINOIS	17	NORTHEAST	20	CORN - ACRES PLANTED	TOTAL	1,056,000
SURVEY	2014	YEAR	AGRICULTURAL DISTRICT	ILLINOIS	17	WEST	30	CORN - ACRES PLANTED	TOTAL	1,147,000
SURVEY	2014	YEAR	AGRICULTURAL DISTRICT	ILLINOIS	17	CENTRAL	40	CORN - ACRES PLANTED	TOTAL	1,606,000
SURVEY	2014	YEAR	AGRICULTURAL DISTRICT	ILLINOIS	17	EAST	50	CORN - ACRES PLANTED	TOTAL	1,638,000
SURVEY	2014	YEAR	AGRICULTURAL DISTRICT	ILLINOIS	17	NORTHWEST	10	CORN - ACRES PLANTED	TOTAL	1,999,000
SURVEY	2014	YEAR	AGRICULTURAL DISTRICT	ILLINOIS	17	EAST SOUTHEAST	70	CORN - ACRES PLANTED	TOTAL	1,579,000
SURVEY	2014	YEAR	AGRICULTURAL DISTRICT	ILLINOIS	17	SOUTHWEST	80	CORN - ACRES PLANTED	TOTAL	580,000
SURVEY	2014	YEAR	AGRICULTURAL DISTRICT	ILLINOIS	17	SOUTHEAST	90	CORN - ACRES PLANTED	TOTAL	624,000
SURVEY	2014	YEAR	AGRICULTURAL DISTRICT	ILLINOIS	17	WEST SOUTHWEST	60	CORN - ACRES PLANTED	TOTAL	1,671,000

State/district: https://quickstats.nass.usda.gov/results/3A17F375-B762-37BD-8C03-D581DC8F7A85
County: https://quickstats.nass.usda.gov/results/478D1A7B-E680-3E5E-95E4-9A59F938A256

JAGS Model

```
#### Assume this source saved in C:/Your Directory Name/Your_JAGS_model.R
model{
        for(j in 1:m){       #Looping over counties, m=102 for Illinois

        #Defines `data model'—note-JAGS uses precision
            thetahat[j] ~ dnorm(theta[j],  1/vhat.dir[j])

        #Defines `process model'
            theta[j] ~ dnorm(beta0+beta1*X1[j]+beta2*X2[j], sigma2u.inv)
        }

    ## Priors:
    sigma2u ~ dunif(0, 10^8)
    sigma2u.inv <- pow(sigma2u, -1)      #Again, precision

    beta0~dnorm(0,.000001)               #Again, precision
    beta1~dnorm(0,.000001)
    beta2~dnorm(0,.000001)
}
```

- Note data, process, prior structure from earlier slide
- Note distributions parameterized in terms of precision
- Read into R script as stored R source code or as text string

Online resources http://www.sumsar.net/blog/2013/06/three-ways-to-run-bayesian-models-in-r/

A Pseudo-Code R Script

```r
##### Loading some libraries--assumes functioning JAGS installation
library(rjags)
library(r2jags)

##### Your data import and wrangling  go here
##### We'll actually fit a model scaled by 'Size' (n reports)
thetahat<-DirInd/Size             #### Survey Estimate
vhat.dir<-VarDirInd/Size^2        #### Estimated Survey Variance
X1<-FSA_DICE/Size                 #### FSA data
X2<-test$pcpn.3                   #### NOAA March Precipitation

############### Initialize Model
set.seed(2018); m=102        #### Set seed, define number of counties

#### Initialize Sampler--Plausible initial value
#### for sigma2u based on least squares
init.sig <- (summary(init.lm.coef)$sigma^2)

#### Distinguish data inputs and parameters
jags.data <- list("thetahat","vhat.dir", "X1","X2","m")
jags.params <- c("theta","sigma2u","beta0","beta1","beta2")

jags.inits <- function(){list("sigma2u" = init.sig)}    #### Function for initial value

#### Execute model: assumes JAGS as source code; object returned is an R list object
jags(jags.data , jags.inits , jags.params, "C:/Your Directory Name/Your_JAGS_model.R",
    n.chains = 3, n.iter = 10000, n.burnin = 1000)
```

Analysis of JAGS Model Output

Posterior summaries of parameters–based on 3,000 saved iterates

- Posterior means, standard deviations, quantiles, potential scale reduction factors, effective sample sizes, pD, DIC

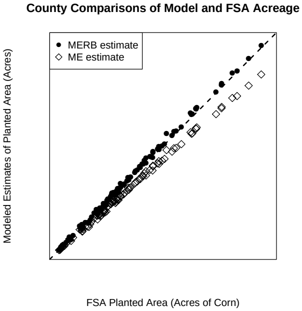
Inference for Bugs model at C:/Your Directory Name/Your_JAGS_model.R
3 chains, each with 10000 iterations (first 1000 discarded), n.thin = 9
n.sims = 3000 iterations saved

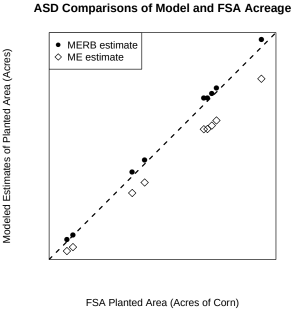
	mu.vect	sd.vect	2.5%	25%	50%	75%	97.5%	Rhat	n.eff
beta0	97.024	205.223	-297.362	-39.365	94.004	235.130	492.579	1.002	1500
beta1	0.865	0.037	0.790	0.841	0.865	0.891	0.937	1.005	830
beta2	-48.553	118.049	-276.194	-126.387	-48.104	28.315	183.179	1.001	2300
sigma2u	20223.038	11544.842	3252.631	11870.939	18247.001	26419.969	47345.031	1.039	84
theta[1]	3399.432	163.965	3083.123	3296.654	3399.326	3505.508	3719.588	1.002	3000
theta[2]	1982.413	153.739	1690.704	1885.191	1977.139	2076.279	2302.119	1.001	3000
theta[3]	2621.446	149.324	2320.691	2525.084	2620.279	2713.351	2925.278	1.001	3000
theta[4]	1296.049	141.511	1014.616	1209.529	1291.823	1383.444	1582.351	1.001	3000
theta[5]	3456.315	157.861	3120.367	3359.261	3458.199	3557.888	3754.838	1.002	1900

- Transform back to acreage scale
- Ratio benchmarking–inject benchmarking factor back into chains as in Erciulescu et al. (2018)

Results: Models With and Without Benchmarking

- Modeled estimates (ME) may not satisfy benchmarking
- Ratio-benchmarked estimates (MERB) are consistent with state targets and improve agreement with external sources

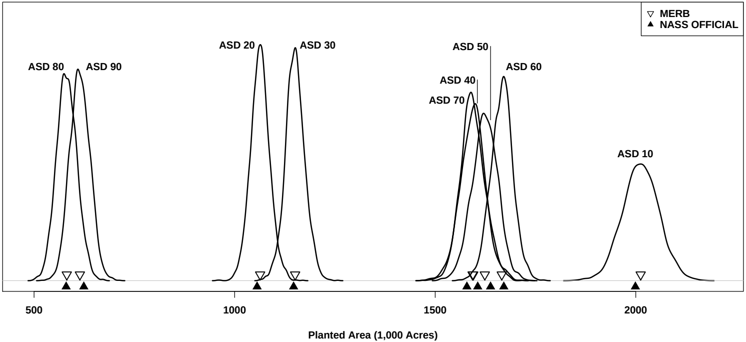




Results: Posterior Distributions of ASD-Level Acreages

Used county-level inputs to produce county-level estimates

- **Idea:** derive ASD-level estimates from Monte Carlo iterates
- Sum corresponding draws from county posterior distributions
  - Compute means and variances from aggregated chains



Results: Relative Variability of Survey Versus Model

Obtain estimates and measures of uncertainty for counties and districts

- Recall the goal of SAE–increased precision!

CV (%) of CAPS Survey Estimates

	Min	*Q1*	*Median*	*Mean*	*Q3*	*Max*
County	9.1	16.6	19.2	22.2	23.5	92.3
District	4.4	5.6	6.8	6.6	7.2	8.7

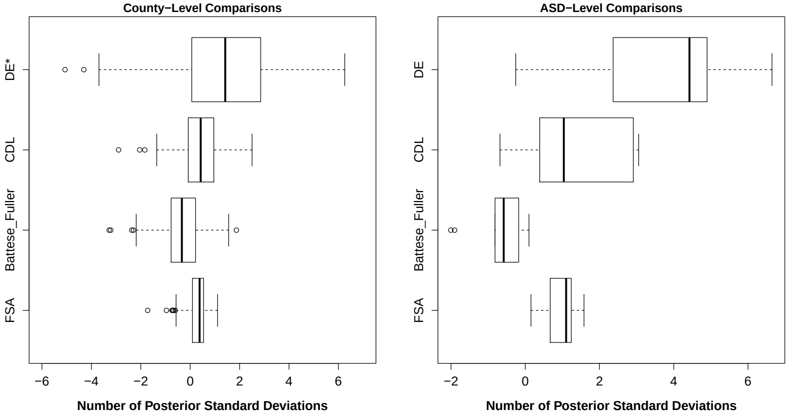
CV (%) of MERB Estimates

	Min	*Q1*	*Median*	*Mean*	*Q3*	*Max*
County	3.6	5.6	7.2	9.0	10.5	31.2
District	1.7	2.0	2.1	2.5	2.3	4.4

Results: Comparison to Other Sources

For counties and districts, compute 'standard score'

- (model estimate-other source)/model standard error
- Direct Estimates, Cropland Data Layer, Battese-Fuller, FSA



Conclusions

Discussed Bayesian formulation of Fay-Herriot model motivated by NASS applications

Other R packages facilitate Bayesian small area estimation

- 'BayesSAE' by Chengchun Shi
- 'hbsae' by Harm Jan Boonstra
- May be bound by limited choice of prior distributions
- Transformations of data may be needed

Proc MCMC in SAS added 'Random' statement as of version 9.3

Thanks to Andreea Erciulescu (NISS) and Balgobin Nandram (WPI) for three years of adventures in small area estimation!

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