Measuring the Services of Property-Casualty Insurance in the NIPAs

Changes in Concepts and Methods

By Baoline Chen and Dennis J. Fixler

As part of the comprehensive revision of the national income and product accounts (NIPAs) that is scheduled to be released on December 10, 2003, a change in the definition of property and casualty insurance services will be introduced. This definitional change will recognize the implicit services that are funded by investment income, will adopt a treatment of insured losses that is more consistent with the economic behavior of the insurer, and will change the treatment of reinsurance. This change is briefly described in the June 2003 issue of the Survey of Current Business, and some of the associated changes in the tables are described in the August 2003 issue.¹

The Bureau of Economic Analysis (BEA) currently measures services of the property-casualty insurance industry as its net premiums earned minus net losses incurred and dividend to policyholders, where net premiums and losses refer to premiums and losses net of reinsurance. However, the insurance output measured using the current definition does not include all the services provided by the property-casualty insurance companies.

Property-casualty insurance companies provide three types of services: Risk-pooling, financial services relating to insured losses, and intermediation. Insurance provides a mechanism for consumers, businesses, and government that are exposed to property-casualty losses to engage in risk reduction through pooling. The insurer provides a variety of real services for policyholders, such as loss settlements, risk surveys, and loss prevention plans. The insurer collects premiums in advance of the loss payments and holds the funds in reserves until the claims are paid. The insurer also provides intermediation services through the investment of the funds in reserves. Net gains from the invested funds in reserves are used to supplement revenue from premiums to pay for claims or for reinsurance services; in other words, policyholders pay a smaller premium in order to compensate for the opportunity cost of their funds that are held by the insurer. According to various studies that focus on the performance of property-casualty insurance services, the provision of these services of financial protection and financial intermediation represents the output of the property-casualty insurance industry (Cummins and Weiss, 2000).

Replacing the actual losses incurred with the normal losses in the calculation of insurance services is a major innovation in the definitional change. Normal losses represent the incurred losses that the insurer expects to pay (payable claims). This change in the treatment of losses recognizes that because actual losses incurred are only known after they occur, insurance companies determine the premiums for an upcoming period on the basis of their perception of the losses that they may incur. The new treatment eliminates the large swings in measured insurance services that are caused by catastrophes, such as the Northridge earthquake in 1994, Hurricane Andrew in 1992, and the terrorist attacks on September 11th, 2001.

Another significant aspect of the definitional change is the use of expected investment income as a measure of premium supplements. Premium supplements are the component of implicit services arising from the investment income earned from the investment in reserves. The inclusion of premium supplements is found in the measure of insurance output in the United Nations System of National Accounts (SNA, 1993), but its inclusion in the BEA measure of output is new. Economic models on the behavior of the insurer generally recognize that insurance companies maximize their profits by setting premiums that are based on their expectations of future losses and investment returns. The use of expected, rather than the actual, investment income to measure premium supplements is intended to better capture the economic behavior of the insurer.

¹ See Moulton and Seskin (2003, 19-23) and Mayerhauser, Smith, and Sullivan (2003, 21).
A much-debated issue about the components of investment income is whether capital gains and the income on own funds should be included. In the SNA, investment income is defined as the interest and dividend income earned on technical reserves, which are the unearned premiums plus unpaid losses. In the estimation of expected investment income, net realized capital gains are included in investment income. Fixler and Moulton (2001) argue that capital gains should be included because the supply price of many services, such as financial services, is based on expected capital gains. Hill (1998) also suggests that capital gains should be treated the same as investment income.

Another issue in the computation of investment income is the treatment of mandated reserves and own-funds. In the United States, the states have regulatory authority over the operations of insurance companies, and in many cases, they mandate the holding of reserves and how the reserves must be invested. Such reserves do not appear as separate entries in the industry consolidated balance sheets. In principle, the invested mandated reserves should be treated the same as other components of technical reserves. Investment income from the insurer’s own-funds is not a component in the premiums supplement and is reported separately from the investment income from the policyholders’ funds, or technical reserves, on the insurer’s annual statement. However, because investment funds are fungible, the industry-level rate of return to invested funds is computed with investment income from both the insurer’s own funds and the policyholders funds.

Currently, insurance services are calculated from data on premiums earned and losses incurred net of reinsurance assumed and ceded. This treatment of reinsurance is based on the assumption that reinsurance services are exports or imports or that reinsurance assumed offsets reinsurance ceded within a particular line of insurance. However, this assumption is incorrect because some domestic insurance companies specialize in reinsurance services and because the data indicate that reinsurance assumed seldom offsets reinsurance ceded within a particular line of insurance. Because insurance companies purchase reinsurance to reduce the risk that they must bear in the event of greater than expected losses, such services will be treated as an intermediate input to the insurance carriers industry or as exports of services.

Under the new definition, services of the property-casualty insurance industry will be measured as direct premiums earned plus premiums supplements minus normal losses incurred and dividends to policyholders. Direct premiums earned equal net premiums earned minus premiums received from reinsurance assumed minus premiums paid for reinsurance ceded. Further discussion on the definitional change and its impact on the national income and product accounts can be found in Moulton and Seskin (2003).

This article discusses the methodology used to incorporate the expectation behavior of the insurer into the insurance output measure. Section 2 focuses on the estimation of normal losses and expected investment income. It describes the expectation behavior of insurers regarding their future losses and their future investment income, and it discusses the statistical methodology for estimating the normal losses and expected investment income. Section 3 discusses the effect of the definitional change on the measured property-casualty insurance services. Section 4 provides the concluding remarks. The article includes a technical note that provides details on the data sources and data preparation for implementing the definitional changes.

Estimation of Normal Losses and Expected Investment Income

To set premiums for a future period, profit-maximizing insurance companies must estimate their expected investment income, their normal losses, and their operating expenses. The importance of expectations is generally accepted, but how expectations on future losses and future investment income are formed is still debatable. Two expectations models that may explain the insurer’s behavior are the adaptive expectations model and the rational expectations model.

In a simple adaptive expectations framework, individuals adjust their expectations according to the deviations of their expectations from their actual experiences. In other words, individuals adapt their expectations according to the forecast errors. Specifically, the expectation for the next period is a weighted average of the actual experience in the current period and the forecast error for the current period. If expressed recursively, the expectation for the next period is a weighted average of the current experience and of all past experiences. The weights on the lagged experiences decline exponentially, emphasizing the importance of the more recent experiences in the formation of expectations.

Adaptive expectations behavior seems consistent with the observation that insurance companies’ estimates of future losses are primarily based on their past

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2. Reinsurance is the purchase of insurance by an insurer. The buyer of the reinsurance is known as the ceding insurer and the seller of the insurance is the assuming insurer.
losses. When evaluating past losses, the insurer accounts for factors, such as the characteristics of the insured, that consistently govern the general behavior of the insured over time toward the insured risks. The insurer also accounts for recent regulatory and technological changes that may have affected recent incurred losses. For example, if there were a recent change in the penalty for drunk driving, then it would likely affect the recent number of accidents caused by drunk drivers. Recent advances in technology in the insurance industry have resulted in better risk surveys and loss prevention programs that are likely to have helped reduce losses. Such factors suggest that more recent loss experiences provide more information about current trends in losses, and hence, more recent loss experiences should carry more weight in the formation of expectations on future losses.

Similarly, current and past investment income provides a major source of information to insurance companies when they estimate investment income for the future periods. However, because other factors, such as the recent performance of the economy or recent changes in tax policy on investment income, may have a more influence on the current trend in investment income, recent investment experiences should be more important in the formation of expectations on future investment income.

The adaptive expectations model is a straightforward way to explain expectations behavior, but Muth (1961) pointed out that this model lacks a theoretical basis, and he proposed a rational expectations framework. Rational expectations theory implies that economic behavior underlies the formation of expectations, and expectations are based on all the information that is available when the expectations are formed. To be consistent with this theory, a structural model that seeks to explain the insurer’s expectations of future losses should include past experiences and variables, such as the prices of materials and services that largely comprise loss payments, number of policyholders, trends in rulings of courts toward legal liabilities, and other variables that may affect future losses. Similarly, in addition to current and past investment income, a structural model that seeks to explain an insurer’s expectation of future investment income should include variables—such as interest rates, the rate of change in technical reserves, the rate of inflation, stock indexes, the rate of growth in real GDP, and other macroeconomic variables—that may affect future investment income.

Rational expectations models are technically difficult to estimate. First, an economic optimization model must be specified, and estimation must be preceded by an analytical solution to the model. Even when the solution is linear in the exogenous variables of the model, the coefficients are often combinations of the structural parameters that are generally not linear and are difficult to estimate. Second, because of the likely serial correlation in the structural disturbances, assumptions about the autocorrelation structure are necessary.

Because of these difficulties, the focus is on the roles of current and past losses and investment experiences, and the adaptive expectations model is used. Despite the theoretical weakness of this model, empirical evidence indicates that it works quite well in many economic applications.

Estimating normal losses or expected investment income is essentially a forecasting problem. Normal losses are future losses that are expected to be paid by the insurer, and hence, statistically, they are the forecasts of future losses. Similarly, expected investment income is the forecast of future investment income. A forecasting method that is consistent with the adaptive expectations framework is a weighted moving average model with weights on the lagged observations declining exponentially. An alternative to this method is the n-point simple moving average method, which has been used by the Australian Bureau of Statistics (1999). Time series methods, such as Autoregressive Integrated Moving Average methods (ARIMA), are also alternatives for forecasting future losses and expected investment income.

A common feature of these methods is that future values of a series depend only on its lagged values.

**Choices of statistical methods**

Under the definitional change, the services of 22 lines of property-casualty insurance is being remeasured. (A list of the 22 lines is included in the technical note.) According to the published records, data series for these lines span from 18 to 72 years. Some lines of insurance services exhibit autocorrelation and possible

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3. Such assumptions are generally arbitrary. Even when a simple autocorrelation structure of the disturbances is imposed, it may not be enough to simplify estimation. Other hypotheses about the autocorrelation function of the structural disturbances may make it impossible to identify the structural parameters or complicate estimation.

4. This weighted moving average method is also known as exponential smoothing or exponentially weighted moving average method. In fact, Muth (1960) shows that if there is no trend and no seasonality, then this model is an autoregressive integrated moving average (ARIMA) model with nonseasonal difference, an MA(1) term, and no constant term, otherwise known as ARIMA(0,1,1). Thus, potentially more sophisticated ARIMA modeling, or Box-Jenkins methods, can be explored.

5. ARIMA methods are developed for estimating concise prediction models of time series data that display complex patterns of autocorrelations.
heteroskedasticity in the residuals in the data series on losses and investment income. Initial experimentation indicated that the search for an optimal ARIMA model to fit the data for each of the 22 lines of property-casualty insurance would be difficult and costly. In addition, to update the estimates annually for each line of insurance when new data become available would add significantly to the costs of producing the national income and product accounts.

The weighted moving average models focus on the trends and seasonal behavior of the data. Because these two aspects largely determine the variance of the series, when chosen properly, the weighted moving average method performs well, relative to more complicated methods, on a wide range of data series. The weighted moving average model with no trend and no seasonal factors requires the estimation of a single parameter. Specifically, the method can be viewed as estimating the value of \( \alpha \) that best fits:

\[
Z_t = w_1 Z_{t-1} + w_2 Z_{t-2} + \ldots + e_t,
\]

where \( w_i = \alpha(1 - \alpha)^{i-1} \), for \( i = 1, \ldots, \) and \( e_t \) is a white noise disturbance term. This formula is identical to that derived from the adaptive expectations model developed by Cagan (1956).

The \( n \)-point simple moving average method is based on the assumption that the time series is “locally stationary” with a slow varying mean. Hence, the moving average of \( n \) most recent observations are used to estimate the current value of the mean, and this mean is used as the forecast for the next period. This method is a compromise between the mean and random walk models. 7

The short-term averaging smooths out the bumps in the original series. By adjusting the degree of smoothing, \( n \), one hopes to strike an optimal balance between the mean and random walk models. The choice for the \( n \)-point average is between a lagged moving average or a centered moving average. The Australian Bureau of Statistics (1999) chooses to use the centered moving averages, implying that the forecast of losses for period \( t \) would be influenced by losses in the future periods. To avoid the influence of future events on the formation of expectations, the lagged moving averages were used for forecasting future losses. 8

Computationally both methods are simple to implement. An advantage of the weighted moving average method is that the small set of model choices simplifies the process of choosing the “best” model and makes it ideal for fairly small data series. The disadvantage of the n-point simple moving average method is that the choice of \( n \) largely depends on subjective judgment because this method is not based on any statistical modeling. The common disadvantage of any moving average method is that the forecasts generated from such a method will lag as the trend of the actual data increases or decreases.

Conceptually, the weighted moving average method is superior to the \( n \)-point simple moving average method because it places relatively more weight on the most recent observations, whereas the \( n \)-point simple moving average method places equal weight on the \( n \) lagged observations and excludes all observations more than \( n \) periods back in time. Moreover, the weighted moving average method relies on a smoothing parameter that is estimated from the entire time series and that is geared toward minimizing the mean square prediction errors.

In order to evaluate the two moving average methods, normal losses and expected investment income for five lines of insurance services were computed, and the summary statistics of the forecast or prediction errors were compared. The five lines of insurance services in the experiment are private passenger auto liability (PAL), private passenger auto physical damage (PAD), homeowners multiple peril (HMP), farmowners multiple peril (FMP), and workers compensation (WCP). These lines were chosen because of their significant shares in the property-casualty insurance industry. In 2000, these five lines accounted for 62 percent of the total premiums earned by the industry, and they accounted for more than 85 percent of the premiums recorded in personal consumption expenditures in the national income and product accounts.

Computing normal losses

The data series that were available for the experiment were direct premiums earned and direct losses incurred from 1972 to 2001. Time series data on direct premiums and losses for almost all the lines of property-casualty insurance services are highly nonstationary. In order to obtain more stationary data and to be able to incorporate information from direct premiums earned, the variable direct losses incurred, \( L_t \), was redefined as the product of direct premiums earned, \( P_t \), and the direct loss ratio, \( L_t = P_t / P_{t-1} \). Thus, the estimates of normal losses were not computed only from direct

6. Autocorrelations summarize temporal persistence of the time series, such as trend, cycle, and seasonal variations.
7. The mean model uses the mean of the entire sample as the estimated value for each period in the sample. The random walk model predicts that one period's value will equal the previous period's value plus a constant representing the average change between periods.
8. In a centered moving average, the estimate for \( t \) depends on values \( t-n/2 \) and values \( t+n/2 \) (with \( n \) being an even number), and the \( t+n/2 \) values would be inconsistent with the estimation of expectations in \( t \).
losses incurred. Instead, expected loss ratios were first estimated from data on direct premiums earned and direct losses incurred, and then estimates of normal losses were derived. Let \( L_{t+1|t} \) be the expected, or the forecasted, loss ratio for period \( t+1 \), given the information available in period \( t \), and let \( L_{t+1} \) be the normal losses for period \( t+1 \). Formally, normal losses in period \( t+1 \) can be expressed as:

\[
L_{t+1} = L_{t+1|t} P_{t+1},
\]

where \( L_{t+1|t} \) is computed as:

\[
L_{t+1|t} = E(L_{t+1|t} | L_t, L_{t-1}, \ldots).
\]

The weighted moving average model discussed above takes the form

\[
E(L_{t+1|t} | L_t, L_{t-1}, \ldots) = \alpha L_t + (1 - \alpha) E(L_{t+1|t} | L_{t-1}, L_{t-2}, \ldots) = \alpha \sum_{i=0}^{\infty} (1 - \alpha)^i L_{t-i},
\]

where \( \alpha \) is the smoothing constant in the interval \((0, 1)\). The expected loss ratio for period \( t+1 \) can be calculated as the weighted sum of the loss ratio at period \( t \) and the forecast of the loss ratio for period \( t \), given information at \( t-1 \). Expressed recursively, the loss ratio at period \( t \) can be calculated as the exponentially weighted sum of loss ratios of all previous periods.

The smoothing parameter, \( \alpha \), can be estimated fairly well if a data series has at least 30 observations and is free of serial correlation. The WinRATS-3.2 Version 5.1 program was used to estimate \( \alpha \), which chooses the estimate of \( \alpha \), \( \hat{\alpha} \), by minimizing in-sample, one-step forecast errors. However, if the data series is not long enough or if it exhibits serial correlation, then setting \( \alpha \) to a reasonable value produces more reliable results than relying on imprecise estimates. According to the statistical and engineering literature, the value of \( \alpha \) is often chosen between 0.1 and 0.3. Some studies point out that an estimated value of \( \hat{\alpha} \) greater than 0.3 may suggest serial correlation in the data series.

Estimating normal losses with the weighted moving average model involves two steps. The first step is to estimate \( \alpha \) and to generate forecasts of loss ratios. If the estimated value of \( \alpha \), \( \hat{\alpha} \), does not suggest serial correlations in the data, then \( \hat{\alpha} \) is used to generate forecasts of loss ratios, \( L_{t+1|t} \). If \( \hat{\alpha} \) indicates serial correlations in the data, then \( \alpha \) is chosen in the interval \((0.1, 0.3)\) to generate loss ratio forecasts, and the chosen \( \alpha \) value, \( \alpha \), is the one with the minimum root mean square prediction errors (RMSE).\(^{10}\) One may experiment with many values of \( \alpha \) in the specified range. The results with \( \alpha = (0.1, 0.2, 0.3) \) indicate that these three choices are sufficient. The second step is to compute normal losses, \( L_{t+1} = L_{t+1|t} P_{t+1} \), and the summary statistics of the in-sample, one-step forecast errors.

In the experiment, estimation results suggest that \( \hat{\alpha} = (0.34, 0.19) \) for HMP and FMP, respectively. For PAL, PAD, and WCP, \( \hat{\alpha} \) indicates serial correlation in the data, so the value of \( \alpha \) was set. Based on the minimum RMSE criterion, \( \alpha = 0.3 \) was set for PAL, PAD, and WCP.

The n-point simple moving average method is straightforward to implement. The expected loss ratio for period \( t+1 \) is given by:

\[
E(L_{t+1|t} | L_t, L_{t-1}, \ldots, L_{t-n+1}) = \frac{1}{n} \sum_{i=0}^{n-1} L_{t-i}.
\]

The main concern with this method is the choice of \( n \). An optimal \( n \) should smooth out the bumps in the data that are generated by short-term noise but still preserve the dynamic characteristics of the time series. However, there is little discussion in the literature on the criterion for choosing an optimal \( n \), perhaps because the n-point moving average method is not based on a formal statistical model.

For the comparison of the two types of moving averages, \( n = 5 \) was selected for each line of insurance. This selection is consistent with the choice of \( \alpha \) because four of the five lines of insurance in the experiment are either 0.3 or close to 0.3, implying that the first five lagged loss ratios account for more than 83 percent of the forecasted loss ratios. An added consideration is that the Australian Bureau of Statistics sets \( n = 5 \) for its forecasts of future losses.\(^{11}\)

Using either moving average method, the estimation of expected losses requires a plan for handling catastrophic losses. By definition, these catastrophes are unpredictable events that have significant effects on losses. Some of the five lines of insurance that were examined have experienced catastrophic losses. For example, homeowners multiple peril (HMP) experienced catastrophic losses in 1992 because of Hurricane Andrew, and the loss ratio for 1992 reached 1.24. Unless adjusted for, catastrophic losses can have too much influence on the computation of expected losses and measured output. Accordingly, the following steps were taken to dampen the

\(^{10}\) Root mean square prediction error is the square root of the average of the squared differences between the actual values and the predicted values for the sample period.

\(^{11}\) BEA’s international transactions accounts recently adopted a 6-year moving average because of the particular features of their data series. (Bach 2003).
effect of catastrophic losses. First, the expected loss ratios using the sample data were computed, and the data for the year of the catastrophe were treated as missing observations. Second, the catastrophic loss was computed as the difference between the actual loss ratio and the estimated loss ratio. Third, the catastrophic loss was spread forward equally for 20 years, starting from the catastrophic year. For example, for HMP, using the weighted moving average method, the adjustment for the catastrophic loss is computed as \( \Delta l = (l_{1992} - l_{1992|1991}(\hat{\alpha})) / 20 \), and using the n-point moving average method, the adjustment is computed as \( \Delta l = (l_{1992} - l_{1992|1991,...,1987}(n = 5)) / 20 \). The adjustment for catastrophic losses, \( \Delta l \), is then added to the forecasts of loss ratios for 1992 through 2011.

In Table 1, the RM SPE for the lines of insurance in the experiment with \( \alpha = (0.1, 0.2, 0.3) \) is compared with the RM SPE for those lines with \( n = 5 \). Note that if \( \alpha \) can be estimated as in the cases of HMP and FMP, \( l_{1,1-5}(\hat{\alpha}) \) yields the minimum RM SPE. If \( \alpha \) cannot be estimated as in the cases of PAL, PAD and WCP, \( l_{1,1-5}(\hat{\alpha} = 0.3) \) yields the smallest RM SPE of all the choices of \( \alpha \) values. The weighted moving average method out performed the 5-point moving average method in four of the five cases.

To further compare the two moving average methods, Table 2 provides the summary statistics that are often used to measure the performance of forecasts: Mean error (ME), mean absolute error (MAE), standard deviation of prediction error (SDPE), and root mean square of prediction error (RMSPE).

Since positive deviations tend to offset negative deviations, MAE is often used to measure the accuracy of the forecasted time series values, in addition to ME that measures the average forecasting error. MAPE is a unit free measure of the accuracy of the forecasts; it converts deviations in any unit measurement to average percentage deviations. SDPE measures the dispersion of the forecast errors, and RMSPE accounts for both the mean and the dispersion of the forecast errors.

For each line, the summary statistics from the forecasts were compared, using the weighted moving averages and choosing \( \alpha \) based on the minimum RM SPE criterion. Summary statistics were also computed from the forecasts using the 5-point moving averages. Columns 2 and 3 contain the summary statistics from the forecasts of loss ratios and Columns 4 and 5 contain

### Table 1. Root Mean Square Prediction Errors (RMSPE) from Loss Ratio Forecasts, Using Weighted and 5-point Moving Averages

<table>
<thead>
<tr>
<th>( \alpha = \hat{\alpha} )</th>
<th>Private auto liability</th>
<th>Private auto physical damage</th>
<th>Homeowners multiple peril (( \hat{\alpha} = 0.34 ))</th>
<th>Homeowners multiple peril (( \hat{\alpha} = 0.19 ))</th>
<th>Workers compensation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = 0.1 )</td>
<td>7.77</td>
<td>6.09</td>
<td>8.74</td>
<td>9.98</td>
<td></td>
</tr>
<tr>
<td>( \alpha = 0.2 )</td>
<td>6.40</td>
<td>6.01</td>
<td>8.42</td>
<td>9.14</td>
<td></td>
</tr>
<tr>
<td>( \alpha = 0.3 )</td>
<td>5.44</td>
<td>5.82</td>
<td>7.81</td>
<td>8.03</td>
<td></td>
</tr>
<tr>
<td>( n = 5 )</td>
<td>6.00</td>
<td>4.89</td>
<td>8.07</td>
<td>7.79</td>
<td>10.29</td>
</tr>
</tbody>
</table>

* Indicates the lowest RMSPE in each column. Root mean square prediction error (RMSPE) is the square root of the average squared difference between the actual value and the prediction value for the sample period.

### Table 2. Summary Statistics of Forecasting Errors from Weighted Moving Averages and 5-Point Moving Averages

<table>
<thead>
<tr>
<th></th>
<th>Forecast errors of expected loss ratio (( \hat{\alpha} = 0.3 )) (percent)</th>
<th>Forecast errors of expected loss ratio (( n = 5 )) (percent)</th>
<th>Forecast errors of normal losses (( \hat{\alpha} = 0.3 )) (percent)</th>
<th>Forecast errors of normal losses (( n = 5 )) (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Private passenger auto liability insurance (1972–2001)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ME</td>
<td>0.88</td>
<td>0.11</td>
<td>-308.06</td>
<td>-554.62</td>
</tr>
<tr>
<td>MAE</td>
<td>4.55</td>
<td>4.98</td>
<td>1,863.76</td>
<td>2,274.62</td>
</tr>
<tr>
<td>MAPE</td>
<td>6.30</td>
<td>6.90</td>
<td>6.30</td>
<td>6.90</td>
</tr>
<tr>
<td>SDPE</td>
<td>5.37</td>
<td>6.00</td>
<td>2,538.19</td>
<td>2,849.99</td>
</tr>
<tr>
<td>RMSPE</td>
<td>5.44</td>
<td>6.00</td>
<td>2,556.69</td>
<td>3,001.87</td>
</tr>
<tr>
<td><strong>B. Private passenger auto physical damage (1972–2001)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ME</td>
<td>1.09</td>
<td>-0.66</td>
<td>1,04.07</td>
<td>3.58</td>
</tr>
<tr>
<td>MAE</td>
<td>3.86</td>
<td>2.94</td>
<td>799.08</td>
<td>909.63</td>
</tr>
<tr>
<td>MAPE</td>
<td>5.80</td>
<td>4.65</td>
<td>5.99</td>
<td>5.61</td>
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<tr>
<td>SDPE</td>
<td>5.72</td>
<td>4.85</td>
<td>1,048.72</td>
<td>1,253.66</td>
</tr>
<tr>
<td>RMSPE</td>
<td>5.82</td>
<td>4.89</td>
<td>1,099.66</td>
<td>1,250.66</td>
</tr>
<tr>
<td><strong>C. Homeowners multiple peril (1972–2001)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ME</td>
<td>0.90</td>
<td>-0.20</td>
<td>314.12</td>
<td>212.58</td>
</tr>
<tr>
<td>MAE</td>
<td>5.80</td>
<td>5.80</td>
<td>1,330.28</td>
<td>1,738.07</td>
</tr>
<tr>
<td>MAPE</td>
<td>8.99</td>
<td>10.50</td>
<td>10.10</td>
<td>11.90</td>
</tr>
<tr>
<td>SDPE</td>
<td>7.22</td>
<td>8.07</td>
<td>2,429.86</td>
<td>2,879.64</td>
</tr>
<tr>
<td>RMSPE</td>
<td>7.28</td>
<td>8.07</td>
<td>2,450.86</td>
<td>2,887.48</td>
</tr>
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<td><strong>D. Farmers compensation (1972–2001)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ME</td>
<td>0.14</td>
<td>0.39</td>
<td>8.15</td>
<td>4.03</td>
</tr>
<tr>
<td>MAE</td>
<td>5.84</td>
<td>6.45</td>
<td>49.08</td>
<td>60.42</td>
</tr>
<tr>
<td>MAPE</td>
<td>8.18</td>
<td>9.18</td>
<td>8.18</td>
<td>9.18</td>
</tr>
<tr>
<td>SDPE</td>
<td>7.39</td>
<td>7.78</td>
<td>66.24</td>
<td>74.96</td>
</tr>
<tr>
<td>RMSPE</td>
<td>7.39</td>
<td>7.79</td>
<td>66.74</td>
<td>75.07</td>
</tr>
<tr>
<td><strong>E. Workers compensation (1972–2001)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ME</td>
<td>0.62</td>
<td>0.58</td>
<td>44.65</td>
<td>38.11</td>
</tr>
<tr>
<td>MAE</td>
<td>0.62</td>
<td>0.81</td>
<td>1,592.67</td>
<td>2,166.27</td>
</tr>
<tr>
<td>MAPE</td>
<td>8.55</td>
<td>11.32</td>
<td>8.55</td>
<td>11.32</td>
</tr>
<tr>
<td>SDPE</td>
<td>8.03</td>
<td>10.27</td>
<td>2,346.33</td>
<td>3,005.74</td>
</tr>
<tr>
<td>RMSPE</td>
<td>8.03</td>
<td>10.29</td>
<td>2,346.75</td>
<td>3,005.87</td>
</tr>
</tbody>
</table>

Forecast errors of loss ratio is \( \frac{1}{n-1} \sum_{t=1}^{n} (\hat{\alpha} - \alpha) \).
Forecast errors of normal losses is \( \frac{1}{n-1} \sum_{t=1}^{n} \Delta l_i - \hat{\alpha} \).

ME is mean error of forecasts
MAE is mean absolute error of forecasts
MAPE is mean absolute percentage error of forecasts
SDPE is standard deviation of forecasting errors
RMSPE is root mean square prediction error.
NOTE. L/P is loss ratio, E(L/P) = \frac{l_{t-1}}{t-1} is expected loss ratio computed with weighted moving averages.

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Chart 1. Actual and Expected Loss Ratios

Computing expected investment income

Data on investment income are labeled as “net investment gain on funds attributable to insurance transactions,” and they are included in part II of the insurance expenditure exhibits (IEE) published in the Best’s Aggregate and Averages: Property-Casualty by A.M. Best Company. The net investment gain on funds attributable to insurance transactions by line of insurance is defined as the product of the industry-level rate of return to invested funds and the technical reserves by line of insurance adjusted for uncollected premiums and for the expenses associated with unearned premiums. The net investment income for the current year includes net realized capital gains. The measurement of investment income here is the same as that used in the producer price index for property-casualty insurance from the Bureau of Labor Statistics (BLS).

Insurance companies often analyze their investment experiences on the basis of the investment income to premium ratios. Let \( I_t \) denote the investment income, and let \( i_t = \frac{I_t}{P_t} \) denote the investment income to premiums ratio in period \( t \). For each line of insurance, direct premiums earned plus premiums supplements in period \( t \), \( P_t + \tilde{I}_t \), can be expressed as \( P_t(1 + i_t) \), which corresponds exactly to the price measure used by BLS in the producer price index for property-casualty insurance. Using this characterization allows the BLS index to deflate the measure of the current-dollar insurance output. Let \( i_{t+1|t} \) be the expected investment income to premiums ratio for period \( t+1 \), given the information available in period \( t \), and let \( I_{t+1|t} \) be the expected investment income for period \( t+1 \) given by:

\[
I_{t+1|t} = i_{t+1|t} P_{t+1}.
\]

\[\boxed{\text{(5)}}\]

\[\text{NOTE. L/P is loss ratio, E(L/P) = } \frac{l_{t-1}}{t-1} \text{ is expected loss ratio computed with weighted moving averages.}\]

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In the weighted moving average model, the expected investment income to premiums ratio is computed as:

\[
i_{t+1} = E(i_{t+1} | i_t, i_{t-1}, \ldots) = \beta \sum_{i=0}^{\infty} (1 - \beta)^i i_{t-i},
\]

where \(\beta\) is the smoothing parameter in \((0, 1)\).

Like the experiment on normal losses, PAL, PAD, HMP, FMP, and WCP are included in the experiment on expected investment income. The estimation experiment used data on investment income to premiums ratios by line of insurance for 1978–2000. Data analysis revealed some degree of serial correlation in the data on \(i_t\), for all five lines of insurance, which led to setting \(\beta = (0.1, 0.2, 0.3)\). As shown in table 3, among the choices of \(\beta\), \(\beta = 0.3\) is associated with the minimum RMSPE. Like the computation of normal losses, the experiment included the n-point moving average method with the parameter \(n = 5\). The estimates that used the weighted moving averages with \(\beta = 0.3\) yield smaller RMSPEs than those used the 5-point moving averages for four of the five lines.

To further compare the estimates from both methods, table 4 shows the summary statistics of the forecast errors from the forecasts that used both moving averages. It is evident that using the weighted moving average method with \(\beta = 0.3\) results in smaller M APE and RM SPE for PAD, HMP, FMP, and WCP. To illustrate the estimation results obtained from using both methods, panels 3.1 to 3.5 in chart 3 provided a comparison of the estimated investment income to premiums ratio with the actual investment income to premiums ratios.

Based on the results from the experiment, the weighted moving average method was chosen to compute the expected loss ratios and expected investment income to premiums ratios for all 22 lines of insurance. This method produced better overall estimation results, and it is consistent with the adaptive expectations model, which conceptually better explains the behavior of the insurer than the n-point moving average method. Because autocorrelation is present in the data series on loss ratios and investment income to premiums ratios for most of the 22 lines, \(\alpha = 0.3\) was used in the computation of expected loss ratios, and \(\beta = 0.3\) was used in the computation of expected investment income to premiums ratios for all 22 lines.

### Effects of Definitional Change on Insurance Output

The definitional change in the output measures of the 22 lines of the property-casualty insurance services has resulted in higher average levels of annual output. The increases derive from the inclusion of investment income as premium supplements, but they are also

---

**Table 3. Root Mean Square Prediction Errors (RMSPE) from Expected Investment Income to Premiums Ratios, Using Weighted Moving Averages and 5-Point Moving Averages**

<table>
<thead>
<tr>
<th>Line of Insurance</th>
<th>Private auto liability</th>
<th>Private auto physical damage</th>
<th>Homeowners multiple peril</th>
<th>Farmowners multiple peril</th>
<th>Workers compensation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta = .1)</td>
<td>1.665</td>
<td>.564</td>
<td>.936</td>
<td>.863</td>
<td>4.174</td>
</tr>
<tr>
<td>(\beta = .2)</td>
<td>1.280</td>
<td>.445</td>
<td>.704</td>
<td>.654</td>
<td>3.137</td>
</tr>
<tr>
<td>(\beta = .3)</td>
<td>1.054</td>
<td>*.391</td>
<td>*.589</td>
<td>*.547</td>
<td>*.655</td>
</tr>
<tr>
<td>(n = 5)</td>
<td>1.080</td>
<td>.391</td>
<td>.604</td>
<td>.664</td>
<td>2.675</td>
</tr>
</tbody>
</table>

* Indicates the minimum RMSPE in each column.

---

**Chart 2. Direct Losses and Normal Losses**

- **2.1 PRIVATE AUTO LIABILITY**
- **2.2 PRIVATE AUTO PHYSICAL DAMAGE**
- **2.3 HOMEOWNERS MULTIPLE PERIL**
- **2.4 FARMOWNERS MULTIPLE PERIL**
- **2.5 WORKERS COMPENSATION**

Note: L is direct losses incurred, and NL = \(L_{t-1} - L_t\) is normal losses.

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attributable, to a much lesser extent, to the use of data on the direct basis, which includes data on reinsurance services. The aggregated average annual output of the 22 lines increased 35 percent; 32 percent of this increase is attributable to the inclusion of data on premium supplements, and 3 percent is attributable to the inclusion of data on reinsurance services.

As was expected, the change to normal losses from actual losses and the use of expected investment income rather than the actual investment income as premium supplements did not significantly affect the aggregated output. The increase in the aggregated annual average output amounted to 0.8 percent. In theory, the aggregated average annual output should not be affected at all if the estimation is conducted properly. The reason for the slight effect is that adjustments for some catastrophic losses are allocated to future years. In addition, only the output of some lines are affected by catastrophic losses, but the aggregate measure is not affected.

The definitional change has also resulted in significantly less volatility in the annual output of the insurance lines that experienced catastrophic losses. The reduction in volatility is largely attributable to the use of normal losses rather than actual losses.

To illustrate the effect of the definitional change and

![Chart 3. Actual and Expected Investment Income to Premiums Ratios](image)

<table>
<thead>
<tr>
<th>Table 4. Summary Statistics of Prediction Errors from Expected Investment Income to Premiums Ratio, Using Weighted and 5-Point Moving Averages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecast errors of expected investment income to premiums ratio ((\beta = 0.3))</td>
</tr>
<tr>
<td>ME</td>
</tr>
<tr>
<td>MAE</td>
</tr>
<tr>
<td>MAPRE</td>
</tr>
<tr>
<td>SDPE</td>
</tr>
<tr>
<td>RMSPE</td>
</tr>
</tbody>
</table>


| Forecast errors of expected investment income to premiums ratio (\(\beta = 0.3\)) | Forecast errors of expected investment income to premiums ratio (\(n = 5\)) | Forecast errors of expected investment income to premiums ratio (\(\beta = 0.3\)) | Forecast errors of expected investment income to premiums ratio (\(n = 5\)) |
| ME | 0.238 | 0.060 | 0.196 | 0.109 |
| MAE | 0.418 | 0.475 | 0.454 | 0.444 |
| MAPRE | 0.098 | 0.100 | 0.900 | 0.092 |
| SDPE | 0.051 | 0.599 | 0.510 | 0.554 |
| RMSPE | 0.589 | 0.604 | 0.547 | 0.565 |

E. Workers compensation (1978–2000)

| Forecast errors of expected investment income to premiums ratio (\(\beta = 0.3\)) | Forecast errors of expected investment income to premiums ratio (\(n = 5\)) |
| ME | 1.573 | 1.049 |
| MAE | 2.122 | 2.094 |
| MAPRE | 0.128 | 0.136 |
| SDPE | 2.104 | 2.436 |
| RMSPE | 2.650 | 2.675 |

Forecast errors of expected investment income to premiums ratio is \(i_t - t_{IP} = 1\).

ME is mean error of forecasts  
MAE is mean absolute error of forecasts  
MAPRE is mean absolute percentage error of forecasts  
SDPE is standard deviation of forecasting or prediction errors  
RMSPE is root mean square prediction errors

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using five insurance lines as an example, table 5 presents a comparison of the average annual output using the current definition with that using the new definition, and it also shows a comparison of the volatility in the actual data series with that in the estimated data series.

The standard deviation of a time series measures the volatility of that series, and the ratio of the standard deviations of two series provides the relative volatility of the two series. Column 2 shows the relative volatility in the expected loss ratios to the actual loss ratios, and column 3 shows the relative volatility in the computed normal losses to the actual losses. Two observations can be drawn from columns 2 and 3. First, the expected loss ratios and the normal losses show reduced volatility. Not surprisingly the reduction in volatility is greater for the lines that experienced catastrophic losses. Allied lines had catastrophic losses in 1989, 1992, 1998, and 2001, and homeowners multiple peril had catastrophic losses in 1992. Second, the reduction in volatility in normal losses is less than that in the estimated loss ratios. This is because normal losses are derived as the product of estimated loss ratios and the direct premiums earned. Some volatility in the direct premiums earned has been picked up in the computed normal losses.

Similarly, column 4 shows that the volatility was reduced as a result of using the expected investment income to premiums ratio rather than the actual investment income to premiums ratio. The reduction in volatility is greater for allied lines; in recent years, the investment income for this line has swung down from an average of 3.78 percent in the 1990s to -6.5 percent in 2000 and to -2.3 percent in 2001.

Additional volatility from the data on reinsurance may be added to the measured output by line of insurance. Therefore, comparing the volatility in the output using the current definition with the volatility in the output using the new definitions does not provide accurate information on the effect of using normal losses and expected investment income. In column 5, that effect is measured by the ratio of the standard deviation of output using the new definition to that of output measured with direct losses and actual investment income as premium supplements; clearly, the use of normal losses and expected investment income reduces the volatility in the output.

In column 6 of table 5, the average annual output using the new definition is compared with average annual output using the current definition. The average output increased significantly, ranging from 8.6 percent for private passenger auto physical damage to 73.4 percent for workers compensation. Because the higher average annual output level is largely due to the inclusion of the expected investment income as premium supplements, the output measured using the current definition significantly underestimates the contributions of the financial intermediation services provided by the property-casualty insurance industry. For the lines in table 5, the average expected investment income is 3.1 percent of the direct premiums earned for allied lines, 3.9 percent for homeowners multiple peril, 4.6 percent for private passenger auto liability, 1.9 percent for private passenger auto physical damage, and 7 percent for workers compensation for their respective sample periods.

In addition to analyzing the effects of the change in the definition of insurance services on average annual output and volatility in the estimated data series for the sample period, the effect of the change can also be

### Table 5. Relative Output and Relative Volatility in Actual and Estimated Data

<table>
<thead>
<tr>
<th>Insurance line</th>
<th>Relative volatility of expected loss ratio versus actual loss ratio</th>
<th>Relative volatility of normal losses versus direct losses</th>
<th>Relative volatility of expected versus actual investment income to premiums ratio</th>
<th>Relative volatility of output using new definition versus output using direct losses and actual investment income</th>
<th>Relative output using new definition versus output using current definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allied lines (1951-2001)</td>
<td>0.370</td>
<td>0.635</td>
<td>0.657</td>
<td>0.392</td>
<td>1.205</td>
</tr>
<tr>
<td>Homeowners multiple peril (1951-2001)</td>
<td>0.800</td>
<td>0.970</td>
<td>0.889</td>
<td>0.900</td>
<td>1.258</td>
</tr>
<tr>
<td>Private auto liability (1930-2001)</td>
<td>0.923</td>
<td>0.978</td>
<td>0.966</td>
<td>0.951</td>
<td>1.273</td>
</tr>
<tr>
<td>Private auto physical damage (1930-2001)</td>
<td>0.802</td>
<td>0.986</td>
<td>0.999</td>
<td>0.974</td>
<td>1.086</td>
</tr>
<tr>
<td>Workers compensation (1930-2001)</td>
<td>0.804</td>
<td>0.998</td>
<td>0.888</td>
<td>0.911</td>
<td>1.734</td>
</tr>
</tbody>
</table>

Relative volatility of expected loss ratio versus actual loss ratio is $\frac{\sigma(l_{(t-1)})}{\sigma(l_t)}$.
Relative volatility of normal losses versus direct losses is $\frac{\sigma(l_{(t-1)})}{\sigma(L_t)}$.
Relative volatility of expected versus actual investment income to premiums ratio is $\frac{\sigma(l_{(t-1)})}{\sigma(L_t)}$.
Relative volatility of output using new definition versus output using direct losses and actual investment income is $\frac{\sigma(Y_{(t-1)}^v)}{\sigma(Y_t^v)}$.
Relative output using new definition to output using current definition is $Y_t^v / Y_t^c$.

Where $\sigma(\cdot)$ is the standard deviation of the time series in the parentheses.

- $l_{(t-1)}$ is expected loss ratio
- $l_t$ is direct loss ratio
- $L_{(t-1)}$ is normal losses
- $L_t$ is direct losses incurred
- $Y_{(t-1)}^v$ is expected net investment income to premiums ratio
- $Y_t^v$ is net investment income to premiums ratio
- $Y_{(t-1)}^c$ is output under new definition, $Y_t^c = P_t(1 - d_t - \beta_{(t-1)}) - \beta_{(t-1)}$
- $Y_t^v$ is output computed as $Y_t^v = P_t(1 - d_t - \beta_t) - L_t$
- $Y_t^c$ is output under current definition, $Y_t^c = P_t(1 - d_t) - L_t$
illustrated from the estimates for a particular year as shown in table 6; 1992 and 2001 were selected to illustrate the effects of the definitional change and to demonstrate how the adjustments for catastrophic losses affect the levels and volatility of the estimated series.

Part A of table 6 presents a comparison of the actual data series with the estimated data series and the output measured using the current definition and the new definitions for 5 lines of insurance for 1992. In 1992, Hurricane Andrew caused catastrophic losses in allied lines and homeowners multiple peril. In column 2, the actual direct loss ratios are 1.20 for allied lines and 1.24 for homeowners multiple peril. In column 3, the corresponding estimated loss ratios, however, are 0.68 for allied lines and 0.73 for homeowners multiple peril. The significantly lower estimated loss ratios reflect the combined effects of estimating loss ratios using the weighted moving averages and the adjustments made for the catastrophic losses.

Columns 4 and 5 in part A of table 6 show a comparison of the actual direct losses and the normal losses. Not surprisingly, the relative values of the actual losses to the estimated loss ratios are not equal to the corresponding relative values of the actual losses to the normal losses. For example, the relative values of the actual loss ratios to the estimated loss ratios (dividing column 2 by column 3) are 1.76 for allied lines, 1.70 for homeowners multiple peril, 0.93 for private auto liability, 0.92 for private auto physical damage, and 0.96 for workers compensation. However, the relative values of the direct losses to the normal losses (dividing column 4 by column 5) are 1.77 for allied lines, 1.70 for homeowners multiple peril, 0.92 for private auto liability, 0.92 for private auto physical damage, and 0.97 for workers compensation. The differential relative values of loss ratios and losses are caused by the additional information from direct losses that is included in the computed normal losses.

Columns 6 and 7 present the actual and expected investment income to premiums ratios for the 5 lines. Columns 8 and 9 present a comparison of the measured output using the current definition with the output using the new definition. Using the current definition, catastrophic losses result in negative output for allied lines and homeowners multiple peril.

Qualitatively similar results are shown in part B of table 6 from estimates for 5 lines of insurance for 2001. Aircraft, fire, and allied lines suffered catastrophic losses as a result of the terrorist attacks on September 11th. In addition to the catastrophic losses, allied lines also had an unusual negative investment income in 2001. This example again demonstrates that using normal losses and expected investment income greatly reduces the large swings in measured output. Using the current definition, the measured output for fire insurance is still positive despite the huge catastrophic losses, because the current definition uses premiums earned and losses incurred net of reinsurance. The direct loss ratio of 1.28 and the positive output of fire insurance service measured using the current definition

### Table 6. A Comparison of Actual and Estimated Loss Ratios, Losses, and Investment Income to Premiums Ratios, and Output Measured Using Current Definition and New Definition

**Losses and output measured in millions of dollars**

<table>
<thead>
<tr>
<th>Insurance Line</th>
<th>Loss ratio (percent)</th>
<th>E (loss ratio) (percent)</th>
<th>Direct losses</th>
<th>Normal losses</th>
<th>Investment income to premiums ratio (percent)</th>
<th>E (investment income to premiums ratio) (percent)</th>
<th>Output using current definition</th>
<th>Output using new definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allied lines…………………</td>
<td>1.20</td>
<td>0.68</td>
<td>3,270.55</td>
<td>1,843.43</td>
<td>0.053</td>
<td>0.043</td>
<td>−10.12</td>
<td>953.66</td>
</tr>
<tr>
<td>Homeowners multiple peril…</td>
<td>1.24</td>
<td>0.73</td>
<td>25,535.85</td>
<td>15,043.51</td>
<td>0.062</td>
<td>0.051</td>
<td>−2,865.80</td>
<td>6,545.00</td>
</tr>
<tr>
<td>Private auto liability……</td>
<td>0.73</td>
<td>0.79</td>
<td>40,793.81</td>
<td>44,094.48</td>
<td>0.100</td>
<td>0.096</td>
<td>13,968.88</td>
<td>16,459.86</td>
</tr>
<tr>
<td>Private auto physical damage</td>
<td>0.56</td>
<td>0.81</td>
<td>18,489.04</td>
<td>20,071.33</td>
<td>0.034</td>
<td>0.039</td>
<td>13,763.49</td>
<td>13,966.69</td>
</tr>
<tr>
<td>Workers compensation………</td>
<td>0.81</td>
<td>0.84</td>
<td>30,513.78</td>
<td>31,536.19</td>
<td>0.210</td>
<td>0.141</td>
<td>3,592.90</td>
<td>8,885.36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Insurance Line</th>
<th>Loss ratio (percent)</th>
<th>E (loss ratio) (percent)</th>
<th>Direct losses</th>
<th>Normal losses</th>
<th>Investment income to premiums ratio (percent)</th>
<th>E (investment income to premiums ratio) (percent)</th>
<th>Output using current definition</th>
<th>Output using new definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aircraft…………………</td>
<td>1.82</td>
<td>0.69</td>
<td>2,992.35</td>
<td>1,228.10</td>
<td>0.268</td>
<td>0.093</td>
<td>−144.63</td>
<td>490.96</td>
</tr>
<tr>
<td>Allied lines…………………</td>
<td>2.04</td>
<td>0.74</td>
<td>8,353.86</td>
<td>3,537.76</td>
<td>0.023</td>
<td>0.004</td>
<td>−37.93</td>
<td>510.67</td>
</tr>
<tr>
<td>Fire………………………….</td>
<td>1.28</td>
<td>0.57</td>
<td>7,541.33</td>
<td>3,585.75</td>
<td>0.023</td>
<td>0.055</td>
<td>1,667.15</td>
<td>2,539.81</td>
</tr>
<tr>
<td>Homeowners multiple peril…</td>
<td>0.77</td>
<td>0.68</td>
<td>27,907.08</td>
<td>24,694.45</td>
<td>0.035</td>
<td>0.044</td>
<td>6,638.63</td>
<td>12,638.69</td>
</tr>
<tr>
<td>Workers compensation………</td>
<td>0.81</td>
<td>0.72</td>
<td>35,473.88</td>
<td>25,448.61</td>
<td>0.183</td>
<td>0.020</td>
<td>4,680.39</td>
<td>14,349.13</td>
</tr>
</tbody>
</table>

Loss ratio is \( l_t \), investment income to premiums ratio is \( i_t \), E (loss ratio) is \( i_{t-1} \), E (investment income to premiums ratio) is \( i_{t-1} \), Direct losses is \( L_t \), Output using current definition is \( Y^*_t = P(1 - d_t) - L_t \), Normal losses is \( L_{t-1} \), Output using new definition is \( Y^*_t = P(1 - d_t + i_{t-1}) - L_{t-1} \).
suggests that a significant portion of the unexpected losses in 2001 were recovered from the reinsurance services purchased.

**Future Research**

The objective of the definitional change in the output measure of property-casualty insurance services was to better measure all the explicit and implicit services provided by the insurer. The estimation results demonstrate that the definitional change and the new statistical treatment of losses and premiums supplements have a substantial impact on the measured insurance services.

However, further research should continue in order to improve the statistical methodology. The adaptive expectations framework often works fairly well empirically, but it lacks theoretical justification. Future research should go toward the construction of a structural model that properly explains how the profit-maximizing insurer uses all the information available to form expectations of future losses and future investment income. Because a much longer time series data set for each line of insurance has now been constructed, more sophisticated time series modeling methods that can better handle the autocorrelations in the data and that could provide more robust estimates should be explored.

**Technical Note: Preparing the Data for the Definitional Change**

The new definition of the property-casualty insurance output can be expressed as:

\[ Y_t = P_t (1 + i_{t|t-1} - d_t) - L_{t|t-1} , \]

where \( Y_t \) is output, \( P_t \) is direct premiums earned, \( L_{t|t-1} \) is normal losses, \( i_{t|t-1} \) is expected investment income to premiums ratio, and \( d_t \) is dividend to premiums ratio for period \( t \). Recall that \( L_{t|t-1} = i_{t|t-1}P_t \), and \( i_{t|t-1} \) is the expected direct loss ratio.

Under the current treatment, BEA uses net premiums earned and net losses incurred to measure insurance output. The change in the measure of insurance output requires the use of direct premiums earned and direct losses incurred. Net premiums earned, \( P^N_t \), equals direct premiums earned minus the net purchases of reinsurance, \( P^R_t \), and net losses incurred, \( L^N_t \), equals direct losses incurred minus losses recovered from net purchases of reinsurance, \( L^R_t \). The net purchase of reinsurance is the difference between the reinsurance ceded and the reinsurance assumed. Because published data on the direct basis is unavailable before 1975, the preceding relationships can be used to derive the needed data by using net reinsurance purchases and net premiums earned and losses incurred.

The definitional change in the measure of insurance output affects the following 22 lines of property-casualty insurance services: Aircraft, allied lines, boiler and machine, burglary and theft, commercial auto liability, commercial auto physical damage, commercial multiple peril, earthquake, farmowners multiple peril, fidelity, fire, homeowners multiple peril, inland marine, medical malpractice, ocean marine, other liability, other lines, private passenger auto liability, private passenger physical damage, reinsurance, surety, and workers compensation. The first step in the implementation of the definitional change is to construct a data set that contains the time series data on \( P_t, L_t, P^N_t, L^N_t, R^P_t, R^R_t, i_t, \) and \( d_t \) for each line of insurance.

**Data sources and data problems**

The main source of data are the 1940 to 2002 editions of Best's Aggregate and Averages: Property-Casualty by A.M. Best Company. The time series for direct premiums earned, direct losses incurred, net investment income, and dividends to policyholders for 1975–2001 are extracted from A.M. Best’s database. Data series for years before 1975 are constructed from A.M. Best’s published data.

The first, 1940 edition of A.M. Best’s data on property-casualty insurance services contained cumulative data for 1930–39 by line of insurance. Therefore, the longest span of the published times series is 72 years, from 1930 to 2001. However, data for all 22 lines of insurance for 1930–2001 are not available; some are only available back to the 1950s, and some date back to the 1970s or 1980s. Table 7 displays the year when the data on each of the 22 lines were either first reported by

<table>
<thead>
<tr>
<th>Insurance line</th>
<th>Year data started</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aircraft</td>
<td>1971</td>
</tr>
<tr>
<td>Allied lines</td>
<td>1951</td>
</tr>
<tr>
<td>Boiler and machine</td>
<td>1930</td>
</tr>
<tr>
<td>Burglary and theft</td>
<td>1930</td>
</tr>
<tr>
<td>Commercial auto liability</td>
<td>1930</td>
</tr>
<tr>
<td>Commercial auto physical damage</td>
<td>1930</td>
</tr>
<tr>
<td>Commercial multiple peril</td>
<td>1966</td>
</tr>
<tr>
<td>Earthquake</td>
<td>1986</td>
</tr>
<tr>
<td>Farmowners multiple peril</td>
<td>1973</td>
</tr>
<tr>
<td>Fidelity</td>
<td>1930</td>
</tr>
<tr>
<td>Fire</td>
<td>1951</td>
</tr>
<tr>
<td>Homeowners multiple peril</td>
<td>1955</td>
</tr>
<tr>
<td>Inland marine</td>
<td>1951</td>
</tr>
<tr>
<td>Medical malpractice</td>
<td>1977</td>
</tr>
<tr>
<td>Ocean marine</td>
<td>1951</td>
</tr>
<tr>
<td>Other lines</td>
<td>1973</td>
</tr>
<tr>
<td>Other liability</td>
<td>1930</td>
</tr>
<tr>
<td>Private auto liability</td>
<td>1930</td>
</tr>
<tr>
<td>Private auto physical damage</td>
<td>1930</td>
</tr>
<tr>
<td>Reinsurance</td>
<td>1973</td>
</tr>
<tr>
<td>Surety</td>
<td>1930</td>
</tr>
<tr>
<td>Workers compensation</td>
<td>1930</td>
</tr>
</tbody>
</table>
A.M. Best or when the data became constructible from the available A.M. Best data.

In addition to the various starting years of the time series for the lines of insurance, there are two other general problems with the published data. First, observations in all of the series except net premiums earned are missing for the early years. As shown in table 8, some series have 20 missing observations, and others have as many as 45 missing observations. The data are missing mainly because the data were published in much less detail then. Over time, more detailed data and better quality data have become available.

Second, in the published data, the classification of certain lines of insurance has changed over time. Some lines were initially components of other lines for some years, but later, these lines were reported as separate lines. Alternatively, some separate lines later became components of other lines. The insurance lines that were affected by changes in classification consist of allied lines, boiler and machine, homeowners and farmowners multiple perils, other liability, other lines, commercial and private auto liability and auto physical damage lines.

**Constructing the data set**

Given the problems with the availability and the quality of the data, it is necessary to construct a set of data for each line of insurance for the sample period.

**Direct premiums earned and direct losses incurred**

A.M. Best began to report business on the direct basis in 1992 in the insurance expense exhibit (IEE), part III—allocation to lines of direct business written, in Best's Aggregates and Averages: Property-Casualty, so data for \( P_t \) and \( L_t \) have been available since then.\(^3\) For the years during which these variables were not reported, they must be derived from other data: \( P_t \) can be derived from the relation between net premiums earned and net premiums for net purchase of reinsurance, and \( L_t \) can be derived from the relation between net losses incurred and net losses recovered from the net purchase of reinsurance as follows:

\[
(P_t) = P_t^N + P_t^R, \quad (L_t) = L_t^N + L_t^R.
\]

Thus, if data on reinsurance, net premiums earned, and net losses incurred are available, \( P_t \) and \( L_t \) can be derived for the years before 1975. Unfortunately, a complete data series on net losses incurred and on the by-line data on reinsurance for the years before 1975 are also unavailable. Thus, extrapolation techniques were used to estimate the missing observations in these series.

There are two problems in constructing the complete series of net premiums earned and net losses incurred. First, net loss ratios were not explicitly reported until 1950. Before 1950, A.M. Best reported loss and adjustment expense ratios jointly. Second, before 1971, net premiums earned and net losses incurred were reported on the basis of the stock, mutual, and reciprocal companies.\(^4\) To obtain the by-line total net premiums earned and the total net losses incurred, the three components needed to be summed. However, data on reciprocal companies were available only for 1971 and 1972 and only for allied lines, fire, homeowners multiple peril, other liability, and workers compensation, and the data were available only for 1972 for private auto liability and private auto physical damage. No data on reciprocal companies for the remaining lines were reported. Thus, the net loss ratios for 1930–49 and the net premiums and net losses for the reciprocal companies for 1930–70 need to be extrapolated.

For the stock and mutual companies net loss ratios first became available for 1950; the shares of net loss

### Table 8. Availability of Published Data on Property-Casualty Insurance

<table>
<thead>
<tr>
<th>Variables</th>
<th>Availability of data series</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_t ) and ( L_t )</td>
<td>1992–2001: By-line and industry total data available 1975–1991: By-line and industry total data available, labeled as adjusted direct premiums and adjusted direct losses incurred 1930–1974: Data unavailable at any level</td>
</tr>
<tr>
<td>( P_t^N ) and ( L_t^N )</td>
<td>1930–1972: By-line data available on the basis of stock, mutual, and reciprocal companies 1930–1949: Data on losses available at any level</td>
</tr>
<tr>
<td>( P_t^R ) and ( L_t^R )</td>
<td>1951–1984: Data on industry total reinsurance data available 1930–1951: Data unavailable at any level</td>
</tr>
<tr>
<td>( d_t )</td>
<td>1975–2001: By-line data available 1930–1974: By-line data unavailable 1951–2001: Data on industry average dividend to premiums ratio available</td>
</tr>
<tr>
<td>( i_t )</td>
<td>1992–2001: By-line data on net investment gain on funds attributable to insurance transactions available 1975–1991: By-line data on net investment gains or losses and other income available 1930–1974: By-line data unavailable 1939–2001: Data on industry total net investment gain or loss available</td>
</tr>
</tbody>
</table>

\( P_t \) is direct premiums earned  
\( L_t \) is direct losses incurred  
\( P_t^N \) is net premiums earned  
\( L_t^N \) is net losses incurred  
\( P_t^R \) is net premiums earned from net purchase of reinsurance  
\( L_t^R \) is net losses recovered from net purchase of reinsurance  
\( d_t \) is ratio of dividend to policyholders to direct premiums earned  
\( i_t \) is ratio of net investment income to premiums earned

---

13. For 1975–91, \( P_t \) and \( L_t \) were reported in IEE in part II—allocation to lines of business net of reinsurance under "adjusted direct premiums earned" and "adjusted direct losses incurred." Before 1975, they were not reported at all.

14. A reciprocal company is an entity formed by individuals, called subscribers, who undertake all types of insurance activities.
The net losses incurred for the stock and mutual companies are then approximated as $L_{t}^{SN} = L_{t}^{SN} + L_{t}^{MN}$ and $L_{t}^{MN} = L_{t}^{MN} + L_{t}^{MN}$.

To obtain the total net premiums earned and the total net losses incurred, an approximation of the premiums and losses for the reciprocal companies was made, but data on the reciprocal companies for some lines are available only for 1971 and 1972. For these lines, the 2-year average ratio of the total net premiums earned to the sum of net premiums earned by the stock and mutual companies,

$$\left( \frac{P_{t}^{SN}}{P_{t}^{SN} + P_{t}^{MN}} + \frac{P_{t}^{SN}}{P_{t}^{SN} + P_{t}^{MN}} \right) / 2$$

were computed. Similarly, the 2-year average ratio of the by-line total net losses incurred to the sum of the net losses incurred for stock and mutual companies,

$$\left( \frac{L_{t}^{SN}}{L_{t}^{SN} + L_{t}^{MN}} + \frac{L_{t}^{SN}}{L_{t}^{SN} + L_{t}^{MN}} \right) / 2$$

were computed. These average ratios were then used to extrapolate the total net premiums earned and the total net losses incurred for $t = 1930, ..., 1970$.

$$\hat{P}_{t} = (P_{t}^{SN} + P_{t}^{MN}) \times \left[ \left( \frac{P_{t}^{SN}}{P_{t}^{SN} + P_{t}^{MN}} + \frac{P_{t}^{SN}}{P_{t}^{SN} + P_{t}^{MN}} \right) / 2 \right],$$

$$\hat{L}_{t} = (L_{t}^{SN} + L_{t}^{MN}) \times \left[ \left( \frac{L_{t}^{SN}}{L_{t}^{SN} + L_{t}^{MN}} + \frac{L_{t}^{SN}}{L_{t}^{SN} + L_{t}^{MN}} \right) / 2 \right].$$

For the lines that reported net premiums and net losses from the reciprocal companies only for 1972, the extrapolator is the 1-year ratio of the total premiums (losses) to the sum of the premiums (losses) from the stock and mutual companies. For the other lines, the total premiums and total losses are the sum of the premiums and losses from the stocks and mutual companies.

As pointed out earlier, the by-line data on reinsurance are not available for years before 1984, and the data on industry total reinsurance have only been available since 1951. To use the available industry data, by-line reinsurance data for 1951–74 were approximated by using the industry total reinsurance data and the share of by-line reinsurance of the industry total. Because reinsurance data are available for each line for 1984–2001, the shares of the net premiums for the net purchase of reinsurance and the net losses recovered from the net purchases of reinsurance for each line were computed for 1984–2001. Then the median of each share series was constructed, and the median was used to extrapolate the by-line net premiums for, and net losses recovered from, net purchases of reinsurance. Specifically, for $t = 1951, ..., 1974$,

$$\hat{P}_{t} = P_{t}^{R,i} \times m \left( \frac{R_{i}}{R_{i}} \right), \quad \hat{L}_{t} = L_{t}^{R,i} \times m \left( \frac{R_{i}}{R_{i}} \right),$$

where $i$ and 1 in the superscript index the insurance line and industry total, respectively, and where $m(-)$ is the median of the shares for 1984–2001. The median instead of the 1984 share was used in order to limit the impact of outlier years.

After $\hat{P}_{t}$ and $\hat{L}_{t}$ are computed, equation (T.2) was used to approximate direct premiums earned and direct losses incurred for 1951–74. However, because no data on reinsurance for 1930–50 are available, direct premiums earned and direct losses incurred for 1930–50 were extrapolated. The extrapolator is based on the assumption that direct premiums earned (direct losses incurred) grew at the same annual rate as net premiums earned (net losses incurred) from 1930 to 1950. This assumption implies that for $t = 1930, ..., 1950$, $P_{t}$ and $L_{t}$ can be extrapolated according to

$$\hat{P}_{t} = P_{t}^{N} \times \left( \frac{P_{t}^{N}}{P_{t}^{N}} \right), \quad \hat{L}_{t} = L_{t}^{N} \times \left( \frac{L_{t}^{N}}{L_{t}^{N}} \right).$$

The above description describes the construction of direct premiums earned and direct losses incurred for the insurance lines that did not change classifications over the years. However, the classifications of some lines changed. Some classification changes did not require an adjustment; for example, farmowners multiple peril was included in homeowners multiple peril until 1973, when it became a separate line. On the other hand, some adjustments were necessary before compiling the data.

### Classification Changes and Adjustments

The classification of the following lines changed: Allied lines, boiler and machine, other liability, other lines, commercial and private auto liabilities and physical damage lines. As a result of these changes, some adjustments were made.

#### Allied Lines

Allied fire and extended coverage were
reported as two lines for 1951–70. In 1971, these two lines were combined to form allied lines. To incorporate this change, allied lines for 1951–70 was computed as the sum of these two lines. Before 1992, multiple peril crop and federal flood insurances were included in allied lines, but they have become two separate lines since then. In 1997, glass was excluded from other lines, and it has been included in allied lines since 1997.

**Boiler and machine.** Steam boiler and engine machine were reported as two separate lines of insurance from 1930 to 1939. In 1940, they were combined as boiler and machine. In order to account for this change, boiler and machine for 1930–39 was computed as the sum of these two lines.

**Other liability.** Other liability has been a separate line since 1975. From 1930 to 1974, other liability was included in miscellaneous liabilities, which became a separate line in 1971. From 1930 to 1970, miscellaneous bodily injury and miscellaneous property damage were listed as separate lines, and they jointly covered the liabilities that were later included in miscellaneous liabilities. To account for this change, miscellaneous liabilities for 1930–70 was computed as the sum of miscellaneous bodily injury and miscellaneous property damage.

In 1975, other liability was formed from a major part of miscellaneous liabilities. The remaining part of miscellaneous liabilities coexisted with other liability for 3 years before it ceased to exist. To reflect this change, the average ratios of other liability (OLB) to miscellaneous liabilities (MLB) for 1975, 1976, and 1977 was computed, and then the average ratios were used as the extrapolators to approximate net premiums earned and net losses incurred for other liability. Specifically, for \( t = 1930, \ldots, 1974 \),

\[
\hat{P}^N_{t,OLB} = P^N_{t,MLB} \times \left[ \frac{P^N_{1975,OLB}}{P^N_{1975,MLB}} + \frac{P^N_{1976,OLB}}{P^N_{1976,MLB}} + \frac{P^N_{1977,OLB}}{P^N_{1977,MLB}} \right]/3,
\]

\[
\hat{L}^N_{t,OLB} = L^N_{t,MLB} \times \left[ \frac{L^N_{1975,OLB}}{L^N_{1975,MLB}} + \frac{L^N_{1976,OLB}}{L^N_{1976,MLB}} + \frac{L^N_{1977,OLB}}{L^N_{1977,MLB}} \right]/3.
\]

**Commercial and Private Auto Insurances.** Commercial auto liability, commercial auto physical damage, private auto liability, and private auto physical damage became individual lines in 1972. For 1930–71, data on private and commercial auto insurances were combined in auto liability and auto physical damage. From 1930 to 1970, the two components of auto liability, auto bodily injury and auto property damage, were two separate lines, and the two components of auto physical damage, auto collision and miscellaneous auto lines, were also two separate lines. Thus, for those years, auto liability and auto physical damage are represented by the sum of these components.

In order to separate private auto insurance from commercial auto insurance, the shares of these insurances that were accounted for by private auto liability and private auto physical damage were computed. These private auto shares have two components: The ratio of private auto insurance to total auto insurance, and the ratio of the share of household to total motor vehicle stock in a given year, \( \frac{MVHS_t}{MVVS_t} \), to the share in 1972, \( \frac{MVHS_{1972}}{MVVS_{1972}} \). For example, for \( t = 1930, \ldots, 1971 \), the private share of auto liability for the net premiums earned, \( SP^P_{t,AL} \), is computed as:

\[
SP^P_{t,AL} = \left[ \frac{P^N_{1972,AL}}{P^N_{1972,\text{Pal}}} \right] \times \frac{MVHS_t}{MVHS_{1972}}.
\]

where \( P^N_{1972,AL} \) is the net premiums earned for private auto liability and \( P^N_{1972,\text{Pal}} \) is total premiums for auto liability. The private share of auto liability for net losses incurred is computed similarly. The private auto shares are constructed to adjust the 1972 private auto insurance to total auto insurance ratio by the changes in the relative motor vehicle stock held by the households over time.

The net premiums earned by private auto liability, \( P^N_{1930-72,\text{Pal}} \), for 1930–72, were approximated as the product of \( P^N_{1930-72,\text{PAL}} \) and \( SP^P_{t,AL} \). Specifically, for \( t = 1930, \ldots, 1972 \),

\[
P^N_{t,AL} = SP^P_{t,AL} \times P^N_{t,\text{PAL}}.
\]

Net premiums earned for private auto physical damage, net losses incurred for private auto liability, and private auto physical damage were approximated in the same fashion as the net premiums for private auto liability. The commercial auto share for auto liability (auto physical damage) was computed as 1 minus private auto share for auto liability (auto physical damage). Net premiums and losses of the commercial auto lines were approximated accordingly.

**Other lines.** The other lines category was created in 1973, and it includes a few small lines reported on the annual statement of the property-casualty insurance industry. Since its creation, the components of other lines have changed several times. From 1973 to 1977, other lines consisted of factory mutual, international, reinsurance, and miscellaneous write-ins. Since 1978, it has included credit (initially credit included mortgage guarantee, which became a separate line in 1992). In 1980, reinsurance became a separate line, and glass became a component of other lines until 1997, when it
became a component of allied lines. Factory mutual was eliminated in the mid-1980s. Currently, other lines consists of credit, mortgage guarantee, international, and miscellaneous write-ins.

As a result of these changes in other lines, the only adjustment made was to remove reinsurance from other lines for 1973–1980, because reinsurance was the largest component, and without an adjustment, there would be a sharp decline in the data series for other lines. In addition, separating reinsurance from other lines allowed a complete time series for reinsurance for 1973–2001 to be constructed. A.M. Best reported other lines with and without reinsurance for 1980–82. Using these reports, the shares of reinsurance in other lines were calculated, and the average of the shares was used to extrapolate reinsurance for 1973–79.

Dividends to policyholders
Since 1975, A.M. Best has provided data on dividends to policyholders by line of insurance. From 1975 to 1991, the data were reported on the net basis, and since 1992, the data have been available on both the net basis and the direct basis. A.M. Best also provided data on the average dividends to policyholders as a ratio of premiums earned at the property-casualty insurance industry level since 1951. From 1930 to 1950, data on dividends were not available at any level, so the industry average dividend ratios for 1951–75 were used to approximate by-line dividend ratios for 1930–50.

For 1975–2001, the relationship between the by-line dividend ratios and the industry average dividend ratios appeared to be relatively stable for most of the lines. A simple regression was run for each line, using the log of dividend ratios by line of insurance as the dependent variable and the log of industry average dividend ratios as the independent variable. The estimated coefficient is statistically significant at the 5-percent level for 15 of the 20 lines (the 2 lines, earthquake and medical malpractice, that started after 1975 were excluded). The regression results were then used to project the dividend ratios for 1951–74 for these 15 lines.

The remaining 5 lines are aircraft, farmowners multiple peril, fidelity, surety, and burglary and theft. In terms of premiums earned, these lines are among the smallest, and most of them have fairly low and flat dividend ratios over time. Thus, for these lines, the average dividend ratios for 1975–2001 were used as the approximated dividend ratios for 1951–74.

Unfortunately, no information on dividend ratios for 1930–51 is available. Since dividend to premium ratios account for less than 1 percent for most lines for 1951–74, the by-line average dividend ratio for 1951–74 was used as the approximated dividend ratios for 1930–50.

Premium supplements
A.M. Best's data on net investment income by line of insurance have been available since 1975. For 1975–91, the data were labeled as "net investment gain or loss and other income," and since 1992, the data have been labeled as "net investment gain on funds attributable to insurance transactions." No data on investment gain by line of insurance are available for years before 1975. However, data on industry total "net investment gain or loss and other income" and data on "total assets invested" for 1939–2001 are available. To fill in the gaps in the series on net investment income by line of insurance, the data for 1939–74 were approximated first, using data at the industry level, and then the data for 1930–39 were approximated.

Using the industry total data for 1939–74, the net investment gain by line of insurance was approximated by multiplying the industry-level rate of return by the technical reserves for each line. The industry-level rate of return was calculated by dividing the total net investment gain or loss by the total assets invested, based on the assumption that each line of insurance had the same rate of return as the industry total for that period. This assumption is consistent with the current calculation of the by-line investment income data reported annually in the IEE table in Best's Aggregates and Averages: Property-Casualty.

Technical reserves, the sum of unearned premiums and unpaid losses, are not readily available by line of insurance. A.M. Best provides data on unearned net premiums from 1930, but it does not provide data on unpaid losses before 1984. Therefore, the median of the ratios of unpaid losses to net losses was computed and used to extrapolate the net unpaid losses, \( \hat{L}^{NU}_t \). Specifically, for \( t = 1930, ..., 1974 \),

\[
(T.10) \quad \hat{L}^{NU}_t = L^{N}_t \times m\left(\frac{L^{NU}_t}{L^{N}_t}\right),
\]

where \( m(.) \) is the median of the ratios of unpaid losses to total net losses incurred from 1984 to 2001.\(^{15}\) To be consistent with the current definition of investment funds used in A.M. Best's reports, the technical reserves for year \( t \) were computed as the average of the sum of unearned premiums and unpaid

\(^{15}\) Because a constructed data series on net losses incurred is available for the entire sample period and because data on unpaid losses for 1984–2001 are available, the regression analysis could be considered to project the by-line unpaid losses for 1930–74. This approach was not pursued, because the sample size of 18 for unpaid losses is too small to produce reliable results.
losses in year $t$ and $t-1$. Thus, net investment income for $t = 1939$, ..., 1974 can be approximated as:

$$
(T.11) \quad \hat{I}_t = r_t^I \times \left[ (P_t^{NU} + P_{t-1}^{NU}) + (L_t^{NU} + L_{t-1}^{NU}) \right]/2,
$$

where $r_t^I$ is the industry-level rate of return to invested funds and $P_t^{NU}$ is the unearned net premiums.

No data on net investment income for 1930–39 are available. The by-line investment income data for these years was approximated by multiplying the estimated technical reserves by the estimated industry-level rate of return. Because the industry-level rate of return for 1939–59 was flat, mostly between 2 and 2.5 percent, the average of the industry-level rate of return for that period was used as the estimated industry-level rate of return for 1930–39.

**References**


